# UNIQUE EQUIVALENCE SCALES ESTIMATION AND IMPLICATIONS FOR DISTRIBUTIONAL ANALYSIS

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Equivalence scales are used to adjust income by family size to obtain income distribution measures. Recently, the concept of equivalence scale elasticity has been introduced to characterise the effect that scales have on distribution measures. We produce utility-based equivalence scales that have the property of constant elasticity. By assuming a particular functional form for the scales and that the scales are independent of the base level of utility, we obtain unique equivalence scales. In contrast to previous estimates of utility-based scales, we do not restrict our sample to particular family types. We determine price-dependent scales by estimating a characteristic-dependent almost ideal demand system using quarterly expenditure data from the US Consumer Expenditure Survey and price indices from the US Consumer Price Index. We use our scales and those implicit in the US official poverty thresholds to adjust expenditures and show that these scales have similar effects on inequality measures.

## 1. INTRODUCTION

In the public policy literature there has been an increased interest in the estimation of equivalence scales. By equating measures of economic welfare across households with different characteristics (such as family

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size), equivalence scales are used to adjust poverty thresholds by family size. Equivalence scales are also used to adjust household income figures to obtain income distribution measures.[1] Since different equivalence scales can yield different measures of poverty or inequality, different equivalence scales should be used depending on the policy being analysed. The key issues involved in selecting an equivalence scale include selecting an appropriate measure of economic welfare and choosing the household characteristics, the functional form of the scales and the data set to use to estimate the scales.

In this paper, we calculate a utility-based equivalence scale, which provides the adjustment that must be made to equate households' actual welfare as determined by a utility function (see van der Gaag and Smolensky 1982, Jorgenson and Slesnick 1987, Betson 1990 and Nelson 1993). We determine the equivalence scales by using the cost functions obtained from estimating an almost ideal demand system. Using the results of Blackorby and Donaldson (1991) and Lewbel (1989), we restrict the class of equivalence scales to those that are independent of the reference level of utility. We obtain utility-based scales such that there are no other equivalence scales, which satisfy our assumptions, that are consistent with the demand data. Hence, our assumptions imply that our scales are unique in our particular class of equivalence scales.

For simplicity, we assume that our equivalence scale is given by the product of family size raised to some power and a function of prices and family size (number of adults and children). For this reason, our scale has the property of constant elasticity, i.e. the percentage change in the equivalence scale due to a percentage change in the family size remains constant for all family sizes.

This concept of elasticity with respect to family size suggests that equivalence scales can be viewed as measures of the economies of scale (in the production of economic welfare) within the household.[2] To determine the effect of these economies of scale on income distribution measures, we use our scales to adjust total expenditures (which we use as a proxy for income) by family size and evaluate the distribution of expenditures. For comparison, we also adjust expenditures by equivalence scales which have different elasticities and find that the effect on various inequality measures is minimal. This is due to the U-shape relationship between an inequality measure and elasticity (see Coulter, Cowell and Jenkins 1992).

The characteristics and expenditures data used are from the US Consumer Expenditure Survey for 1987-1991. The price data are price indices from the US Consumer Price Index series for the same time period. The sample upon which we base our analysis is not restricted by household characteristics. (Our sample is only restricted to households with positive total expenditures and income, non-negative health expenditures, and which are complete income reporters).[3] This is in contrast to the approach taken by previous researchers who, for example, restricted their samples to households with persons less than a certain age or to households with children (see, for example, Nelson 1993).

We base our scales on quarterly expenditure data, not annual (or four quarters of) data, again in contrast to the approach followed by previous researchers (see, for example, van der Gaag and Smolensky 1982; Jorgenson and Slesnick 1987 and Nelson 1993). Characteristics of the sample from quarterly data are more like those for the total population than are the characteristics of samples from annual data. For this reason, equivalence scales based on our quarterly data should be more useful than other equivalence scales when adjusting incomes and expenditures for distributional analysis. While our scales are similar to the scales implicit in the US official poverty thresholds, our scales are steeper than other scales obtained using similar utility-based methods (see van der Gaag and Smolensky 1982, Betson 1990 and Nelson 1993).

The second section of this paper describes utility-based equivalence scales and presents the almost ideal demand system models that we use to estimate our equivalence scales. The third and fourth sections present the data used in and the results of our estimation, respectively. The fifth section presents the effect these scales have on inequality measures. The final section concludes the paper and provides directions for future research.

## 2. EQUIVALENCE SCALES AND DEMAND DATA

Utility-based equivalence scales are obtained by determining the income level needed to equate the utility levels of two households with different demographic characteristics. We assume that the household utility functions are given by $u(x,\alpha)$ (where $x$ is a vector of commodities and $\alpha$ is a vector of demographic characteristics), and calculate the equivalence scales using the indirect utility levels, $v(p,m,\alpha)$. We obtain the equivalence scales by determining the income level, $m$, that equates the indirect utility levels of a household with characteristics $\alpha$ and a reference household with characteristics $\alpha_r$ and income $m_r$.

For a given price level, $p$, characteristics, $\alpha$, and utility level, $u$, the equivalence scale, $\Delta$, can be given by $m/m_r$, such that $v(p,m,\alpha) = u$. This implies that the equivalence scale is found by implicitly solving the equation, $v(p,\Delta m_r,\alpha) = v(p,m_r,\alpha_r)$. Since $m_r = C(u,p,\alpha_r)$ (the cost function), we define $\Delta$ in terms of the cost functions, i.e.

$$\Delta(u,p,\alpha;\alpha_r) = \frac{C(u,p,\alpha)}{C(u,p,\alpha_r)} \tag{1}$$

By normalising with respect to $\alpha_r$, we drop the reference to $\alpha_r$ and simply refer to the equivalence scales as $\Delta(u,p,\alpha)$.

By viewing the characteristics, $\alpha$, as family size and the reference household as a one-person household, we can see how these scales can represent the economies of scale within a household. If there were no economies of scale in the household, then a two-person household would require twice the income as the reference household. Assuming that households produce economic welfare (measured with $u(x,\alpha)$) from the consumption of goods, economies of scale are present if the equivalence scale is less than the family size, which is usually the case. By measuring economic welfare this way, we ignore the intrahousehold distribution of resources as well as the use of time in household production.

While these scales give adults and children the same weight, there may be different economies of scales for adults and children. Children could cost less than adults since they consume less of some goods, such as food. On the other hand, other costs of children, such as child care and education, could increase the relative costs of children versus adults, so that children and adults could have the same effect on households costs.

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## 2.1 Equivalence Scale Exactness

A problem with utility-based equivalence scales is that they cannot be identified by examining demand data alone because different reference levels of utility will yield different equivalence scales. Blundell and Lewbel (1991) show that demand data can only identify a relative equivalence scale, which represents the ratio of cost-of-living indices for households with different characteristics. They show that Equation (1) can be represented as the product of a relative scale and an absolute scale, which determines the characteristic changes at base prices, p0. That, (1) can be rewritten as:

$$\Delta(u,p,p_0,\alpha) = \frac{D(u,p,p_0,\alpha)}{D(u,p,p_0,\alpha_r)} \cdot \Delta_A(u,p_0,\alpha) \qquad (2)$$

where $D(u,p,p_0,\alpha) = C(u,p,\alpha)/C(u,p_0,\alpha)$ represents a cost-of-living index for the household with characteristics, $\alpha$, and $\Delta_A(u,p_0,\alpha)$ represents the absolute scale evaluated at base prices, $p_0$. Since different levels of household utility can change the absolute scale and leave the relative scales unchanged, the scales may change depending on the assumptions made about $\Delta_A(u,p_0,\alpha)$.

To deal with the multiplicity of scales, Blackorby and Donaldson (1991) and Lewbel (1989) suggest imposing conditions on the equivalence scale, and hence on preferences, which reduce the number of possible forms the scale can possess. They provide conditions under which the scales are independent of the base level of utility. Following Blackorby and Donaldson (1991), we say that $\Delta(u,p,\alpha)$ satisfies *equivalence scale exactness* (or *independence of base* as in Lewbel (1989)) if and only if the cost function is of the following form:

$$C(u,p,\alpha) = c_1(u,p)c_2(p,\alpha) \tag{3}$$

Recalling that equivalence scales are given by the ratio of the cost functions, (3) implies that the equivalence scale is given by:

$$\Delta(u,p,\alpha) = \Delta(p,\alpha) = \frac{c_2(p,\alpha)}{c_2(p,\alpha_r)} \tag{4}$$

Since the utility level does not enter into Equation (4), the use of different reference utility levels will not change the equivalence scales. In addition, arbitrary scaling of the households' income levels will not change the value of the equivalence scales. As a result, estimates of the equivalence scales from characteristic-specific demand data will be the same for all levels of income and utility.

Condition (4) also shows that prices can affect the scales. In this paper, we estimate price-dependent scales and scales that are independent of prices, which are called Full Engel scales (see Blackorby and Donaldson 1991). By allowing price dependence, the scales in Equation (4) can be viewed as a generalisation of Engel scales.

## 2.2 THE ALMOST IDEAL DEMAND SYSTEM

To obtain equivalence scales, we use demand data to estimate characteristic-dependent cost functions. We modify the almost ideal demand system presented by Deaton and Muellbauer (1980) by allowing the cost function to depend on characteristics. This characteristic-dependent almost ideal demand system is described by the following cost function:

$$\ln C(u,p,\alpha) = \ln a(p,\alpha) + b(p,\alpha)u \tag{5}$$

With this cost function, if $b(\cdot,\cdot)$ is independent of $\alpha$ (i.e. $b(p,\alpha) = b(p)$ for all $\alpha$), then the cost function satisfies condition (3), which implies equivalence scale exactness. In this case, $c_2(p,\alpha) = a(p,\alpha)$ and $c_1(u,p) = \exp(b(p)u)$. As in Deaton and Muellbauer (1980), we assume that $b(p) = \Pi p_i^{b_i}$ and that $\ln a(p,\alpha) = a_0(\alpha) + \Sigma a_i(\alpha)\ln p_i + 0.5\,\Sigma\,\Sigma\,c_{ij}\ln p_i \ln p_j$. For the system to be homogeneous (of degree zero in prices) and symmetric, we make the following restrictions:

Homogeneity restrictions:

$$\sum_i c_{ij} = \sum_j c_{ij} = 0, \quad \sum b_i = 0, \quad \text{and} \sum a_i(\alpha) = 1 \tag{6a}$$

for all $\alpha$, and

Symmetry restrictions:

$$c_{ij} = c_{ji}. \tag{6b}$$

By normalising the scale so that $a_0(\alpha_r) = a_0$ and $a_i(\alpha_r) = a_i$, we obtain the following cost function in terms of $a(p,\alpha)$, $b(p)$ and $\Delta(p,\alpha)$:

$$\ln C(u,p,\alpha) = a_0 + \sum a_i \ln p_i + \tfrac{1}{2} \sum \sum c_{ij} \ln p_i \ln p_j + u \cdot \prod p_i^{b_i} + \ln \Delta(p,\alpha). \tag{7}$$

Using (7), the logarithmic form of Shepherd's lemma implies that the share for good $i$ can be given by:

$$w_i = a_i + \sum_j c_{ij} \ln p_j + b_i\left(\ln \frac{m}{a(p,\alpha_r) \cdot \Delta(p,\alpha)}\right) + \frac{\partial \ln \Delta(p,\alpha)}{\partial \ln p_i}. \tag{8}$$

Following Deaton and Muellbauer (1980) and Pashardes (1991), we use a Stone index to approximate $a(p,\alpha)$.[4] We assume that $a_0 + d_0(\alpha) + \ln P_h$ such that $P_h$ is given by:

$$\ln P_h = \sum a_i \ln p_i + \tfrac{1}{2} \sum c_{ij} \ln p_i \ln p_j + \sum d_i(\alpha_h) \ln p_i, \tag{9}$$

we use a household-specific Stone index, $P_h^* = \Sigma\, w_{ih} \ln p_i$, to approximate $P_h$. Substituting the Stone index into the share equations in (8) and dropping the subscript h yields:

$$w_i = a_i - b_i a_0 + \sum c_{ij} \ln p_j + b_i(\ln \frac{m}{P^*}) - b_i \ln \Delta(p,\alpha) + \frac{\partial \ln \Delta(p,\alpha)}{\partial \ln p_i}. \tag{10}$$

### 2.3 The Equivalence Scales

Given our assumptions about the structure of $a(p,\alpha)$ and recalling that $\Delta(p,\alpha) = a(p,\alpha)/a(p,\alpha_r)$, our equivalence scales are given by:

$$\Delta(p,\alpha) = \exp(d_0(\alpha)) \cdot \prod p_i^{d_i(\alpha)}. \tag{11}$$

These equivalence scales are similar to those in Blackorby and Donaldson (1991), Nelson (1993) and Phipps (1991).

Blackorby and Donaldson (1991) show that with the almost ideal demand system different scales that satisfy equivalence scale exactness can be consistent with the same demand behavior. They show that there are functions of characteristics and prices which when multiplied by the scale yield new scales while leaving the demand behavior unchanged. Since the first component in Equation (11) is independent of prices and the second component is Cobb-Douglas in prices, equivalence scale exactness and demand data yield unique equivalence scales.

In our estimation, we assume that characteristics, $\alpha$, are given by the number of adults ($A$) and the number of children ($K$) in the household, i.e., $\alpha = (A,K)$. To capture economies of scale (and constant elasticity), we assume that $d_0(\alpha)$ depends only on the total number of people in the household, i.e. $d_0(\alpha) = d_0 \ln(A + K)$. Assuming that $d_i(\alpha) = \Sigma d_{ik}\alpha_k$, our equivalence scales are given by:[5]

$$\Delta(p,\alpha) = (A+K)^{d_0} \cdot \prod p_i^{(d_{ih}A + d_{ik}K)} \tag{12a}$$

In (12a), the parameter $d_0$ represents constant elasticity. A value of one implies no economies of scale, since the equivalence scale is approximately equal to the family size, $(A + K)$. We would expect the estimate of $d_0$ to be between zero and one.

The scale in (12a) also illustrates the difference between the absolute and relative scales. In (12a), the absolute scale can be represented by the first component, $(A + K)^{d_0}$, and the relative scales can be obtained using the second component. Given prices, $p^1$ and $p^0$, for any number of adults and children, the relative scale is given by $\Pi(p_i^1/p_i^0)d_{ih}(A - 1) + d_{ik}K)$.

To check the importance of allowing the equivalence scale to depend on prices, we also estimate an equivalence scale that is independent of prices. This Full Engel equivalence scale is given by:

$$\Delta(p,\alpha) = (A+K)^{d_0}. \tag{12b}$$

To check the constant elasticity assumption, we estimate another Full Engel scale that allows different family sizes to have different effects on the size of the scale. The Full Engel scale without constant elasticity is given by:

$$\Delta(p,\alpha) = \exp\left(d_{02}(fs_2) + d_{03}(fs_3) + d_{04}(fs_4) + d_{05}(fs_5) + d_{06}(fs_6)\right) \tag{12c}$$

where $fs_i = 1$ for a family of size $i$ and $fs_6 = 1$ for families of size six or more.

Substituting the scales given by (12a) into the share equations of the almost ideal demand system yields:[6]

$$w_i = a_i - b_i a_0 + (d_{ih} \cdot A + d_{ik} \cdot K) - b_i(d_0 \ln(A+K)) + \sum c_{ij} \ln p_j + b_i(\ln \frac{m}{P*}). \tag{13}$$

Equation (13) shows that $(d_0 \ln(A + K))$ enters each share equation, suggesting that the scales deflate the household's income, $m$, in the same manner in each share equation. This is in contrast to Barten-type scales, which are share-specific since they allow the characteristics to affect income differently in each share equation. The difference between the effects of the price-dependent scales and the Full Engel scales can also be illustrated with Equation (13). Since the price-dependent scales include the terms, $(d_{ih} \cdot A + d_{ik} \cdot K)$, the share equations can respond differently to different family sizes. By excluding these terms, the Full Engel scales require family size to affect each share equation in the same manner.

The parameter $b_i$ in Equation (13) is independent of characteristics, that is, family size and income enter the share equation separately and there are no family size-income interaction terms. This shows equivalence scale exactness, which means that all households will have the same coefficients on income in their demand equations.[7]

## 3. THE DATA

The expenditure and consumer unit characteristics data are from the US Consumer Expenditure Survey Interview portion (CEX) for 1987 quarter two through 1991 quarter one, in which time 84187 CEX interviews were conducted (USDL 1991b). We analyse quarterly expenditure data from the internal Bureau of Labor Statistics (BLS) files (data are not topcoded).

We reduced the sample to 71593 by including only consumer units designated by the BLS as complete income reporters and having positive values for total expenditures and income, and non-negative health expenditures. We further reduced the sample by selecting a five per cent random sample to limit the costs of running the models. Our random sample includes 3550 consumer units; 5.8 per cent of these are households for which expenditures are observed for more than one quarter. Descriptive statistics of the sample are presented in Table 1.[8]

We use expenditures for consumption only; expenditures for gifts and capital improvements, and allocations to savings are excluded. We grouped expenditures into seven commodity groups: food, shelter, apparel and upkeep, transportation, fuels and utilities, health care, and ‘other’, which includes household furnishings and operations, entertainment, and other goods and services.

Price data are area-specific price indices from the Consumer Price Index (CPI) listed on the BLS LABSTAT data system for the same time period as the expenditure data (USDL 1992). The base period for the indices was 1982 to 1984. We used the seasonally unadjusted consumer price indices in thirty areas for the first six commodity groups, while a weighted average price was computed for the `other' expenditures commodity group; the weights were the shares of total expenditures allocated to the three commodity groups included in the `other' category.

We used price indices from fifteen urban areas, while regional and citysize average price indices were used for the other areas represented in the

**Table 1**
**Descriptive Statistics**
**($n$ = 3550; 1987–1991)**

| Variable | | Mean | Standard Deviation |
|---|---|---|---|
| Total Quarterly Expenditures | | $5877.85 | 906.77 |
| Shares: | Food | 0.200 | 0.105 |
| | Shelter | 0.306 | 0.143 |
| | Apparel | 0.049 | 0.056 |
| | Transportation | 0.159 | 0.155 |
| | Fuel | 0.097 | 0.064 |
| | Health | 0.056 | 0.075 |
| | Other | 0.134 | 0.114 |
| Prices: | Food | 122.154 | 7.839 |
| | Shelter | 129.352 | 12.877 |
| | Apparel | 117.060 | 8.645 |
| | Transportation | 112.387 | 6.878 |
| | Fuel | 108.213 | 7.701 |
| | Health | 145.289 | 13.362 |
| | Other | 124.880 | 7.378 |
| Stone Index (log) | | 4.808 | 0.070 |
| Family Size | | 2.604 | 1.526 |
| Log of Family Size | | 0.788 | 0.589 |
| Frequency of Family Size | | | |
| | 1 989 | 0.279 | — |
| | 2 1033 | 0.291 | — |
| | 3 588 | 0.166 | — |
| | 4 542 | 0.153 | — |
| | 5 242 | 0.068 | — |
| | 6 90 | 0.025 | — |
| | 7 36 | 0.010 | — |
| | 8+ 30 | 0.008 | — |
| Number of adults | | 1.868 | 0.812 |
| Frequency of adults | | | |
| | 1 1177 | 0.332 | — |
| | 2 1863 | 0.525 | — |
| | 3 355 | 0.100 | — |
| | 4 121 | 0.034 | — |
| | 5+ 34 | 0.010 | — |
| Number of children | | 0.736 | 1.133 |
| Frequency of children | | | |
| | 0 2193 | 0.618 | — |
| | 1 558 | 0.157 | — |
| | 2 493 | 0.139 | — |
| | 3 205 | 0.058 | — |
| | 4 67 | 0.019 | — |
| | 5+ 34 | 0.010 | — |

sample. To obtain quarterly prices, we averaged monthly prices (or bimonthly, if monthly prices were not available) over the three months of the interview quarter.

While the area-specific prices used in this paper do not provide a measure of price differences across areas, they do provide an accurate measure of price differences over time within areas and we show that they do not bias the parameter estimates in the equivalence scales. We tested whether these area-specific prices bias the results by estimating a fixed effects model which included a dummy variable for each area in each equation. We found that, while the price coefficients are different, the presence of fixed effects do not significantly improve the explanatory power of the model and hence, we cannot reject the hypothesis that the fixed effects were insignificant.[9] To further check the possible bias caused by area-specific prices, we estimate the model using average national price indices (for each area) and obtain similar parameter estimates of the characteristic variables. While our results suggest that using area-specific prices do not bias the parameter estimates, in future research we plan to use new BLS research in which interarea prices are produced (see Kokoski, Cardiff and Moulton 1992).

## 4. THE RESULTS

We estimated Equation (13) for six share equations; the coefficients for the seventh equation (other) are calculated using the homogeneity and symmetry restrictions. Tests of the symmetry restriction (6b) allowed us to reject this restriction, while tests of the homogeneity restrictions (6a) did not allow us to reject this restriction. The estimates were obtained using the non-linear seemingly unrelated regression procedure in SAS.

The parameter estimates, standard errors and $R^2$ values for the almost ideal demand systems using three different functional forms of the scales are given in Table 2. As the R2 values suggest, the model with pricedependent scales fits the data better than the models without these terms. This suggests that price changes affect different size households differently.

Equivalence scales are calculated by substituting the parameter estimates (from Table 2) and the average prices for the data set (over all four years) into Equations (12a,b and c). The equivalence scales and their approximate standard errors are given in Table 3. We normalised the scales so that a single-member household has an equivalence scale of one. Except for two-person households, the price-dependent scales are larger (although not significantly) than both of the Full Engel scales.

**Table 2**
**Almost Ideal Demand System Estimation**
**Using Different Functional Forms for the Equivalence Scales**

| Parameter | Variable | Parameter Estimate by Model (Standard Error) | | |
|---|---|---|---|---|
| | | Price-dependent Scale | Full Engel Scale with Constant Elasticity | Full Engel Scale with Family Size Dummies |
| $A_i$ | Food | 0.428[1] | 0.438[1] | 0.441[1] |
| | | (0.012) | (0.012) | (0.012) |
| | Shelter | 0.392[1] | 0.346[1] | 0.332[1] |
| | | (0.014) | (0.013) | (0.013) |
| | Apparel | 0.054[1] | 0.060[1] | 0.061[1] |
| | | (0.006) | (0.006) | (0.006) |
| | Transportation | −0.166[1] | −0.134[1] | −0.126[1] |
| | | (0.017) | (0.016) | (0.016) |
| | Fuel | 0.196[1] | 0.195[1] | 1.096[1] |
| | | (0.007) | (0.006) | (0.006) |
| | Health | 0.058[1] | 0.046[1] | 0.045[1] |
| | | (0.010) | (0.010) | (0.010) |
| | Other | 0.038 | 0.049 | 0.051 |
| | | — | — | — |
| $b_i$ | Food | −0.078[1] | −0.078[1] | −0.080[1] |
| | | (0.002) | (0.002) | (0.002) |
| | Shelter | −0.020[1] | −0.018[1] | −0.014[1] |
| | | (0.004) | (0.004) | (0.004) |
| | Apparel | −0.001 | −0.003[2] | −0.003[2] |
| | | (0.001) | (0.001) | (0.001) |
| | Transportation | 0.101[1] | 0.102[1] | 0.100[1] |
| | | (0.004) | (0.004) | (0.004) |
| | Fuel | −0.029[1] | −0.029[1] | −0.029[1] |
| | | (0.002) | (0.002) | (0.002) |
| | Health | −0.003 | 0.001 | 0.001 |
| | | (0.002) | (0.002) | (0.002) |
| | Other | 0.030 | 0.025 | 0.025 |
| | | — | — | — |
| $c_{ji}$ | Food–Food | 0.065 | 0.061 | 0.058 |
| | | (0.083) | (0.084) | (0.084) |
| | Food–Shelter | 0.012 | 0.015 | 0.017 |
| | | (0.025) | (0.025) | (0.025) |
| | Food–Apparel | −0.029 | −0.032 | −0.032 |
| | | (0.025) | (0.025) | (0.025) |
| | Food–Transportation | −0.018 | −0.020 | −0.021 |
| | | (0.055) | (0.055) | (0.055) |
| $c_{ij}$ | Food–Fuel | −0.007 | −0.002 | −0.001 |
| | | (0.022) | (0.022) | (0.022) |
| | Food–Health | 0.035 | 0.041 | 0.039 |
| | | (0.038) | (0.039) | (0.039) |
| | Food–Other | −0.058 | −0.063 | −0.060 |
| | | — | — | — |

*(Table 2 continued overleaf)*

*(Table 2 continued)*

| Parameter | Variable | Parameter Estimate by Model (Standard Error) | | |
|---|---|---|---|---|
| | | Price-dependent Scale | Full Engel Scale with Constant Elasticity | Full Engel Scale with Family Size Dummies |
| | Shelter-Shelter | 0.280' | 0.272' | 027I' |
| | | (0.032) | (0.032) | (0.032) |
| | Shelter Apparel | 0.038' | 0.039' | 0.039' |
| | | (0.014) | 10.014) | (0.014) |
| | Shelter Transportation | -0.125' | -O.I19' | -0.121' |
| | | (0.034) | (0.034) | (0.034) |
| | Shelter-Fuel | -0.115' | -0.118' | -0.1172 |
| | | (0.014) | (0.014) | (0.014) |
| | Shelter-Health | -0.121' | -0.122' | -0.122' |
| | | (0.019) | (0.019) | (0.019) |
| | Shelter-Other | 0.031 | 0.033 | 0.033 |
| | Apparel-Apparel | -0.016 | -0.017 | -0.017 |
| | | (0.017) | (0.017) | (0.017) |
| | Apparel Transportation | 0.002 | 0.003 | 0.003 |
| | | (0.027) | (0.028) | t0.028) |
| | Apparel-Fuel | 0.001 | 0.002 | 0.002 |
| | | (0.011) | (0.011) | (0.011) |
| | Apparel-Health | -0.033 | -0.031 | -0.031 |
| | | (0.017) | (0.017) | (0.017) |
| | Apparel-Other | 0.037 | 0.036 | 0.036 |
| | Transportation- Transportation | - 0.150 | - 0.144 | - 0.148 |
| | | (0.087) | (0.087) | (0.087) |
| | Transportation-Fuel | 0.080' | 0.082' | 0.08(?' |
| | | (0.027) | (0.027) | (0.027) |
| | Transportation-Health | 0.007 | 0.006 | 0.007 |
| | | (0.039) | (0.039) | (0.039) |
| | Transportation-Other | -0.096 | -0.0% | -0.096 |
| | Fuel Fuel | -0.021 | -0.023 | -0.022 |
| | | (0.015) | (0.015) | (0.015) |
| | Fuel-Health | 0.020 | 0.016 | 0.016 |
| | | (0.016) | (0.016) | (0.016) |
| | Fuel-Other | 0.042 | 0.043 | 0.042 |
| | Health-Health | 0.087' | 0.091' | 0.091' |
| | | (0.035) | (0.035) | (0.036) |
| | Health-Other | 0.005 | -0.001 | 0.000 |
| | Other-Other | 0.039 | 0.048 | 0.045 |
| *do* | Family Size | 0.667' | 0.567 | |
| | | (0.086) | (0.025) | |
| | Family Size 2 | – | - | 0.475' |
| | | | | (0.038) |

| Parameter | Variable | Parameter Estimate by Model (Standard Error) | | |
|---|---|---|---|---|
| | | Price-dependent Scale | Full Engel Scale with Constant Elasticity | Full Engel Scale with Family Size Dummies |
| | Family Size 3 | — | — | 0.645[1] (0.045) |
| | Family Size 4 | — | — | 0.751[1] (0.046) |
| | Family Size 5 | — | — | 0.954[1] (0.062) |
| | Family Size 6+ | — | — | 1.092[1] (0.075) |
| $d_{ii}$ | Food — No. of Adults | 0.001 (0.003) | — | — |
| | Shelter — No. of Adults | −0.016[1] (0.003) | — | — |
| | Apparel — No. of Adults | −0.003[1] (0.001) | — | — |
| | Transportation — No. of Adults | 0.018[1] (0.005) | — | — |
| | Fuel — No. of Adults | 0.0003 (0.002) | — | — |
| | Health — No. of Adults | 0.004[2] (0.002) | — | — |
| | Other — No. of Adults | −0.0043 — | — | — |
| $d_{ik}$ | Food — No. of Children | 0.003 (0.003) | — | — |
| | Shelter — No. of Children | −0.019[1] (0.002) | — | — |
| | Apparel — No. of Children | 0.006[1] (0.001) | — | — |
| | Transportation — No. of Children | 0.010[1] (0.003) | — | — |
| | Fuel — No. of Children | −0.003[2] (0.001) | — | — |
| | Health — No. of Children | −0.010[1] (0.001) | — | — |
| | Other — No. of Children | 0.013 — | — | — |
| Adjusted $R^2$ for share equations | Food | .245 | .240 | .238 |
| | Shelter | .055 | .023 | .024 |
| | Apparel | .020 | .005 | .005 |
| | Transportation | .186 | .177 | .178 |
| | Fuel | .106 | .105 | .107 |
| | Health | .031 | .011 | .010 |

[1]Statistically significant at the 1% level.
[2]Statistically significant at the 5% level.

**Table 3**
**Equivalence Scales**

| Number of Adults | Number of Children | Equivalence Scale (Approximate Standard Error): Price-Dependent Scale | Full Engel Scale with Constant Elasticity | Full Engel Scale with Family Size Dummies |
|---|---|---|---|---|
| 1 | 0 | 1.000 | 1.000 | 1.000 |
| 2 | 0 | 1.585 | 1.481 | 1.608 |
| | | (0.060) | (0.037) | (0.048) |
| 2 | 1 | 2.070 | 1.864 | 1.906 |
| | | (0.094) | (0.046) | (0.056) |
| 2 | 2 | 2.500 | 2.194 | 2.119 |
| | | (0.119) | (0.054) | (0.063) |
| 2 | 3 | 2.890 | 2.491 | 2.596 |
| | | (0.138) | (0.061) | (0.077) |
| 2 | 4 | 3.253 | 2.762 | 2.981 |
| | | (0.154) | (0.068) | (0.088) |
| 1 | 1 | 1.582 | 1.481 | 1.608 |
| 1 | 2 | 2.067 | 1.864 | 1.906 |
| 1 | 3 | 2.495 | 2.194 | 2.119 |
| 1 | 4 | 2.885 | 2.491 | 2.596 |

### 4.1 Constant Elasticity

The parameter $d_0$ is a measure of the overall economies of scale due to family size and accounts for most of the differences in the scales across family size. The estimates of $d_0$ are .67 in the price-dependent scales, .57 in the Full Engel scales with constant elasticity, and an approximated value of .58 in the Full Engel scales with dummy variables. These results suggest that there are economies of scale in the production of economic welfare.

We tested the assumption of constant elasticity by including dummy variables for family sizes three through six and seven or more in addition to the log of family size. We could not reject the hypothesis that all of the coefficients on the dummy variables were zero. This implies that different family sizes do not change the constant elasticity parameter.

Table 3 shows that the Full Engel scales with and without constant elasticity are almost identical, which suggests that the constant elasticity assumption is not overly restrictive. Constant elasticity and the fact that most of the families of size three or more consist of not more than two adults suggests that adults and children do not have different effects on the overall economies of scale, that is, it is the overall family size that is important in determining the economies of scale.

### 4.2 Price-characteristic Interaction Terms

We also tested whether the price-characteristic interaction terms significantly contribute to the explanatory power of the model and we were able to reject the hypothesis that these coefficients were zero. While the price-characteristic variables are significant, the homogeneity restrictions imply that the equivalence scales will not change significantly as prices change (see Phipps 1991).

Not only do the price-characteristic interaction terms improve the fit of the model, their inclusion causes the constant elasticity parameter to increase. This could be due to the fact that the interaction terms allow the share equations to depend on the characteristics in different ways. We would expect increases in family size to cause a greater increase in the food share than in all of the other shares since there are less economies of scale in food expenditures. As Table 2 shows, the $d_0$ parameter is larger in the price-dependent model and the interaction terms in the food share are close to zero and they are negative in other share equations.

While the constant elasticity parameter represents the absolute scale, price-characteristic interaction terms are used to determine the relative scales. Taken together, the price-characteristic terms yield relative scales that are less than one for households with children and greater than one for all-adult households. A negative coefficient on an interaction term implies that as the relative price of the commodity increases, the equivalence scale for larger households decreases, because it becomes less costly to live in a large household.

As expected, the coefficients in the shelter share equation are negative for both adults and children. The coefficients in the transportation equation are positive suggesting that increases in the relative price of transportation represent a higher cost for large households than for small households, perhaps reflecting the fact that the average number of cars per household increases with family size.

The positive coefficient for the interaction term between children and apparel supports the results presented in Nelson (1989) which suggest that children are ‘clothing-purchase intensive.’ The negative coefficient for the interaction term between children and health may reflect the pricing structure of health plans in the US, with family health insurance policies usually covering children no matter how many are present.

### 4.3 Equivalence Scale Exactness

Finally, we tested the assumption of equivalence scale exactness. Recent attempts at estimating equivalence scales that satisfy equivalence scale exactness have found this assumption to be overly restrictive; Blundell and Lewbel (1991) and Pashardes (1992) both reject the hypotheses of equivalence scale exactness. We obtained similar results.

We test for equivalence scale exactness by estimating a model is which we allowed the coefficients on income to depend on family size. The resulting share equations depended on a new term, the product of family size and income. We found these terms to be significant in each share equation and were able to reject the hypothesis that the parameters were all zero. This suggests that total expenditures and family size are correlated and that interaction terms should be included in the share equations. Future work will involve estimating a Rank-3 demand system, which allows for this interaction while still allowing equivalence scale exactness (see Pashardes 1994).

## 5. EQUIVALENCE SCALES AND DISTRIBUT10N MEASURES

In this section we examine the impact of different equivalence scales on the distribution of expenditures, which are used as a proxy for income. We compare three different measures of inequality over the entire sample. For these comparisons, we inflate the expenditures by the CPI so that all expenditures are in 1990 dollars. We adjust the expenditures by family size using the equivalence scales implicit in the US official poverty thresholds and by those produced from the estimations of the price-dependent model and the two Full Engel models. For comparison, we also present results in which expenditures are not adjusted for family size (household weighted) and in which expenditures are simply divided by family size (per capita).

The relationship between equivalence scale elasticities and inequality indices are shown in Table 4. Results are presented for three distributional measures; two are from the Generalised Entropy family of indices, the Theil and the Mean Logarithmic Deviation, and the other is the Gini coefficient.

Coulter, Cowell and Jenkins (1992) show the effect of changing the elasticity of the equivalence scale on various inequality indices. They show that the Theil and Mean Logarithmic Deviation inequality indices plotted against the equivalence scale elasticity yields a U-shaped curve. In other words, for small (close to zero) and large (close to one) elasticities, the inequality indices will be higher than for elasticities in the one-half range. This U-shape is also exhibited for the Gini coefficient for some distributions.

Our results using the US CEX confirm the U-shape relationship for the

**Table 4**
**Distributional Measures**
**by Method to Adjust Expenditures by Family Size**

| Expenditure Adjustment | Scale Elasticity | Inequality Measure: Gini | Inequality Measure: Thiel | Inequality Measure: Mean Logarithmic Deviation |
|---|---|---|---|---|
| Household Weighted | 0.00 | .372 | .246 | .244 |
| Per Capita | 1.00 | .388 | .271 | .261 |
| Poverty Scale | 0.55 | .358 | .230 | .222 |
| Price Dependent Scale | 0.67 | .358 | .231 | .220 |
| Full Engel Scale with Constant Elasticity | 0.57 | .354 | .226 | .215 |
| Full Engel with Family Size Dummies | 0.58 | .353 | .225 | .213 |

three inequality indices. Table 4 shows that the indices are greatest when the expenditures are not adjusted by family size and when the per capita adjustment is used. The elasticities obtained in our models are in a fairly fiat segment of the U-shaped relationship (see Cowell, Coulter and Jerkins 1992) and hence, the differences in the inequality indices are minimal. Our equivalence scale estimates yield inequality indices that are also similar to those obtained by adjusting expenditures with the US poverty scales.

## 6. CONCLUDING REMARKS

Using expenditure data and price indices to estimate an almost ideal demand system, we produced three different equivalence scales. By assuming a particular functional form for the scales and that the scales are independent of the base level of utility, we obtained unique equivalence scales. We showed that price effects should be included in the equivalence scales and that the scales have the property of constant elasticity. In general, our scales are larger than earlier estimates using similar utilitybased methods (see, Nelson 1993 and Betson 1990).

Even though the elasticity of the scales differ, their impact on three measures of inequality is minimal. Since our scales are similar to the scales used in adjusting the US official poverty thresholds by family size, the difference in the inequality measures was also small. In our scales (as in the poverty scales), adults and children are given the same weight. Since there may be different economies of scale for adults and children as suggested by the estimates of the price-characteristic terms, we plan to investigate the separate effects of children and adults and to compare the equivalence scales. Another area for future research is to compare single-parent and two-parent households.

In future research, we will continue our exploration of the methodological issues related to the estimation of equivalence scales. Specifically, we plan to test whether the use of interarea prices or a longer sample period will produce more price variation. Since other researchers have used restricted samples, we plan to use our model on similarly restricted samples to determine the sensitivity of sample selectivity. We also plan to further examine the implications of requiring equivalence scale exactness and particular functional forms. A complete analysis would examine different methods of obtaining equivalence scale exactness in an almost ideal demand system, including the translog form of the scales as presented in Jorgenson and Slesnick (1987), the form in Pashardes (1991) and a Rank-3 demand system as in Pashardes (1994).

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## NOTES

1. See US House of Representatives 1991, Table 26, p. 1184.
2. Buhmann et al. (1988) introduce the concept of constant elasticity with respect to family size to classify the various equivalence scale estimates. Ruggles (1990) also suggests using this type of scale.
3. See USDL (1991a) for a definition of complete income reporter.
4. Even though Pashardes (1993) shows that the Stone index biases the price coefficients, we can show that the Stone index does not bias the equivalence scale estimates.
5. The homogeneity restrictions (6a) require that $\Sigma a_i(\alpha) = 1$ for all $\alpha$, and hence $\Sigma a_i = 1$ and $\Sigma d_i(\alpha) = 0$, which implies that $\Sigma d_{ih} = 0$ and $\Sigma d_{ik} = 0$.
6. We only estimate an intercept term, $A_i = a_i - b_i a_0$, and do not identify $a_0$. To check the influence of $a_0$, we estimate the model with different values of $a_0$ to show that only the $A_i$ parameters change; all other parameter estimates remain the same.
7. Barten scales have goods-specific scales, $\Delta_i$, in which case, $\ln b(p,\alpha) = \Sigma b_i (\ln p_i - \ln \Delta_i(\alpha))$. Barten scales satisfy equivalence scale exactness if $\Sigma b_i (\ln \Delta_i(\alpha)) = 0$ (see Lewbel 1989), which holds in our model since $\Delta_i = \Delta_0$ for all $i$.
8. See Johnson and Garner (1993) for a detailed description of the data set.
9. A comparison of the parameter estimates with and without fixed effects as well as with average national prices are available from the authors.

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