An Introductory Look at the Chained Consumer Price Index

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ESTIMATION METHODOLGY

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1. Price Index Formula Notation

Index symbol

Abbreviation of underlying price index formula

Item, area, and population represented

$$\underbrace{{}_{i,a;u}}_{\text{Item, area, and population represented}} IX^{L}_{[0;t]}$$

Base period of index

Comparison period of index

2. The Cost-of-Living Index Concept

$${}_iIX^{C}_{[0;t]} = \frac{\min \sum_i {}_iP_t \times {}_iQ_t \Big|_{U=U_0}}{\sum_i {}_iP_0 \times {}_iQ_0}$$

- The minimum expenditure (P_t•Q_t) required in comparison period (t) to attain the same level of satisfaction or utility (U_0) achieved in base period (0), divided by the actual expenditure (P_0•Q_0) in base period (0).

3. Price Index Formulas Commonly Used to Approximate a Cost-of-Living Index

FIRST ORDER APPROXIMATIONS:

LASPEYRES:	$IX^{L}_{[0;t]} = \sum_i {}_is_0 \left(\frac{{}_ip_t}{{}_ip_0} \right)$
PAASCHE:	$IX^{P}_{[0;t]} = \left[\sum_i {}_is_t \left(\frac{{}_ip_0}{{}_ip_t} \right) \right]^{-1}$
GEOMETRIC MEAN:	$IX^{G}_{[0;t]} = \prod_i \left(\frac{{}_ip_t}{{}_ip_0} \right)^{{}_is_0}$

SECOND ORDER APPROXIMATIONS:

TORNQVIST:	$IX^{T}_{[0;t]} = \prod_i \left(\frac{{}_ip_t}{{}_ip_0} \right)^{\left(\frac{{}_is_0 + {}_is_t}{2} \right)}$
FISHER IDEAL:	$IX^{F}_{[0;t]} = \left(IX^{L}_{[0;t]} \times IX^{P}_{[0;t]} \right)^{1/2}$

<u>KEY</u>:

${}_ip_t$	=	Price of item (i) in comparison period (t)
${}_ip_0$	=	Price of item (i) in base period (0)
${}_is_t$	=	Expenditure on item (i) in comparison period (t), divided by expenditures on all items in comparison period (t)
${}_is_0$	=	Expenditure on item (i) in base period (0), divided by expenditures on all items in base period (0)

4. Estimation of Price Change in the Chained Consumer Price Index (C-CPI-U)

LOWER-LEVEL AGGREGATION:

$$ {}_{i,a}IX^{L}_{[0;t]} = \sum_{k\in i,a} {}_{k}s_{0} \left(\frac{{}_{k}p_{t}}{{}_{k}p_{0}} \right) \quad \textbf{or} \quad IX^{G}_{[0;t]} = \prod_{k\in i,a} \left(\frac{{}_{k}p_{t}}{{}_{k}p_{0}} \right)^{{}_{k}s_{0}} $$

UPPER-LEVEL AGGREGATION:

	Long-term Price Change	Month-to-Month Price Change
Initial C-CPI-U	${}_{I,A}IX^{Gi}_{[z;y,t]} = {}_{I,A}IX^{Gr}_{[z;y-1,12]} \times \prod_{n=1}^{t} {}_{I,A}IX^{Gi}_{[n-1;n]}$	${}_{I,A}IX^{Gi}_{[t-1;t]} = l_{y} \prod_{i,a\in I,A} \left(\frac{{}_{i,a}IX^{LorG}_{[0;t]}}{{}_{i,a}IX^{LorG}_{[0;t-1]}} \right)^{{}_{i,a}S_{b_1}}$
Interim C-CPI-U	${}_{I,A}IX^{Gr}_{[z;y,t]} = {}_{I,A}IX^{T}_{[z;y-1,12]} \times \prod_{n=1}^{t} {}_{I,A}IX^{Gr}_{[n-1;n]}$	${}_{I,A}IX^{Gr}_{[t-1;t]} = l_{y} \prod_{i,a\in I,A} \left(\frac{{}_{i,a}IX^{LorG}_{[0;t]}}{{}_{i,a}IX^{LorG}_{[0;t-1]}} \right)^{{}_{i,a}S_{b_2}}$
Final C-CPI-U	${}_{I,A}IX^{T}_{[z;y,t]} = {}_{I,A}IX^{T}_{[z;t-1]} \times {}_{I,A}IX^{T}_{[t-1;t]}$	${}_{I,A}IX^{T}_{[t-1;t]} = \prod_{i,a\in I,A} \left(\frac{{}_{i,a}IX^{LorG}_{[0;t]}}{{}_{i,a}IX^{LorG}_{[0;t-1]}} \right)^{\frac{{}_{i,a}S_{t-1} + {}_{i,a}S_{t}}{2}}$

<u>KEY</u>:

- k = unique good or service
- A = CPI aggregate area
- a = CPI elementary area
- I = CPI aggregate item
- i = CPI elementary item
- 0 = elementary index base period
- t = month
- y = year
- ${}_{k}p_{t}$ = price of good (k) in month (t)
- ${}_{k}p_{0}$ = price of good (k) in base-period (0)
- ${}_{k}s_{0}$ = expenditure for good (k) in base period (0), divided by expenditure for all (k) goods in elementary item (i), area (a) in base period (0)
- z = December 1999 index base period
- b_i = expenditure reference period of CPI-U index of year (y); NOTE: b_i = 1999-2000 for y=2002 and y=2003.
- b_r = expenditure reference period of CPI-U index of year (y+1). NOTE: b_r=b_i for y=2002 and b_r=2001-2002 for y=2003.
- IX^{L} = Laspeyres elementary index
- IX^{G} = Geometric Mean elementary index
- IX^{Gi} = Initial C-CPI-U index
- IX^{Gr} = Interim C-CPI-U index
- IX^{T} = Final C-CPI-U index
- ${}_{i,a}S_{bi}$ = expenditure for elementary item (i) in area (a) in expenditure period (b_i), divided by expenditure for all elementary items in aggregate item (I) in aggregate area (A) in expenditure period (b_i)
- ${}_{i,a}S_{br}$ = expenditure for elementary item (i) in area (a) in expenditure period (b_r), divided by expenditure for all elementary items in aggregate item (I) in aggregate area (A) in expenditure period (b_r)
- ${}_{i,a}S_{t}$ = expenditure for elementary item (i) in area (a) in month (t), divided by expenditure for all elementary items in aggregate item (I) in aggregate area (A) in month (t)
- ${}_{i,a}S_{t-1}$ = expenditure for elementary item (i) in area (a) in month (t-1), divided by expenditure for all elementary items in aggregate item (I) in aggregate area (A) in month (t-1)
- λ_{y} = Adjustment factor used in year (y) to calculate Initial (y) and Interim (y-1) C-CPI-U indexes published in year (y); NOTE: λ_{y}=1 for C-CPI-U indexes published in 2002.