ALLOCATING SAMPLE TO STRATA PROPORTIONAL TO AGGREGATE MEASURE OF SIZE WITH BOTH UPPER AND LOWER BOUNDS ON THE NUMBER OF UNITS IN EACH STRATUM

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1. Introduction

Consider the following common sample design. A sample of n units is to be selected from a frame consisting of M units that is partitioned into H strata, with M_h units in strata h. The units within each stratum are to be selected with probability proportional to size, without replacement. Let T_{hi}, $h = 1,...,H$, $i = 1,...,M_h$, denote the measure of size (MOS) for unit i in stratum h; let $T_h = \sum_{i=1}^{M_h} T_{hi}$ denote the aggregate MOS for stratum h; and let $T = \sum_{h=1}^{H} T_h$. A common method of allocating the sample among the strata is proportional to the aggregate MOS. That is, if n_h denotes the number of sample units allocated to stratum h, then

$$n_h = n\frac{T_h}{T} \quad (1.1)$$

There are two problems associated with (1.1). First, it does not generally yield an integer-valued allocation, that is, some form of rounding is required of the allocations in (1.1). We will not focus on this problem. The other problem is that we must have

$$n_h \le M_h \text{ for all } h \quad (1.2)$$

However, the allocation given by (1.1) does not necessarily satisfy (1.2). The standard approach to handling this problem (Cochran 1977, Sec. 5.8) is to

$$\text{reallocate } n_h = M_h \text{ for all } h \text{ for which } n_h > M_h \quad (1.3)$$

and then

reallocate the remaining sample to the remaining strata proportional to T_h (1.4)

However, the new allocation to the remaining strata still may not satisfy (1.2) for all the strata, in which case this process of fixing the sample size at M_h for all strata for which $n_h > M_h$ and reallocating the remaining sample to the remaining strata proportional to T_h is repeated until (1.2) is satisfied for all strata.

To illustrate consider Table 1. (In all of the tables, $n = 72$, and $H = 10$.) For the initial allocation given in the fourth column, (1.2) is violated for stratum 1 since $n_1 = 40.91$ and $M_1 = 9$. Therefore, for the second allocation we let $n_1 = 9$ and reallocate the remaining 63 units to the other 9 strata proportional to T_h. (Those strata whose sample size is fixed at M_h are indicated in bold.) Since (1.2) is violated for stratum 2 for the second allocation, we let $n_2 = 10$ for the third allocation. For the fourth allocation, the sample sizes for strata 3, 4, and 5 are additionally fixed at their maximum values. The fourth allocation is the final unrounded allocation since (1.2) is then satisfied for all strata. In the next column we obtain an integer-valued allocation by rounding up a sufficient number of the unrounded values with the largest fractional remainders to preserve the sample total of 72 and rounding down the remaining values. This is only one of a number of rounding methods discussed in Balinski and Young (1982).

The final allocation before rounding obtained through this recursive process is as close as possible to being proportional to the aggregate MOS given the constraints (1.2) in the following sense. There is a common ratio $r = n_h / T_h$ for all strata h for which $n_h < M_h$, while

$$n_h / T_h \le r \text{ for all } h \text{ for which } n_h = M_h \quad (1.5)$$

In this sense the final allocation is optimal. To illustrate, (1.5) holds for the final unrounded allocation in Table 1 with $r = 0.0012$. The final values of n_h / T_h are given in the last column of the table with the values in bold for those h for which $n_h = M_h$.

Similarly, suppose a lower bound, m_h, is placed on the sample size for each stratum h and it is still desired to allocate proportional to T_h as closely as possible subject now to the constraints

$$n_h \ge m_h \text{ for all } h \quad (1.6)$$

Then, if the initial allocation (1.1) does not satisfy (1.6) but does satisfy (1.2), an analogous recursive algorithm can be used in which we repeatedly

$$\text{reallocate } n_h = m_h \text{ for all } h \text{ for which } n_h < m_h \quad (1.7)$$

and then use (1.4). If (1.2) holds for the initial allocation, it will also hold for every subsequent allocation in the recursion, since the allocation is continually being lowered for all h for which $n_h \geq m_h$. Hence, there is no need to reallocate to satisfy the upper bounds. Consequently, the recursive algorithm used to satisfy (1.6) will yield an allocation as close as possible to being proportional to the aggregate MOS given the constraints (1.6) in the sense that there will be a common ratio $r = n_h / T_h$ for all strata h for which $n_h > m_h$ and

$$n_h / T_h \geq r \text{ for all } h \text{ for which } n_h = m_h \quad (1.8)$$

This situation is illustrated by Table 2. (Those strata with sample size fixed at m_h are italicized in the tables as is the final value of n_h / T_h for each such stratum.) Here three iterations are needed and $r = 0.00038$ for the final allocation.

Next, what if the initial allocation violates (1.2) for some strata and (1.6) for other strata? It might appear that, analogously to the previous situations, we would use a recursive process where at each iteration after the first we would reallocate to the former set of strata using (1.3) and the later set of strata using (1.7), and then use (1.4). However, that algorithm does not yield a final allocation that generally meets the desired criteria that there is a common ratio

$$r = n_h / T_h \text{ for all } h \text{ for which } m_h < n_h < M_h \quad (1.9)$$

and that (1.2), (1.5), (1.6), and (1.8) all hold.

To illustrate, consider Table 3. Here for each iteration we reallocated using (1.3) and (1.7). It required four iterations to satisfy (1.2) and (1.6). However, although (1.9) holds for the final allocation in this table with $r = 0.0014$ and (1.5) also holds, (1.8) is violated for $h = 3,5,7,8$.

In Table 4 we present a different approach to the same example that does satisfy all of the conditions (1.2), (1.5), (1.6), (1.8), and (1.9). Here in the second iteration we reallocated using (1.3), that is let $n_1 = 9$, and then used (1.4) without applying (1.7) first. In iteration 3 we repeated this process. However, in iteration 4 we reallocated using (1.7) but not (1.3). In iteration 5 we used (1.3) only and finally in iteration 6, (1.7) only. Since the allocation given by iteration 6 satisfies (1.2) and (1.6) we stop. Then for this final allocation (1.9) is satisfied with $r = 0.0011$, and (1.5) and (1.8) also hold.

Note in Table 3, which did not work, we applied both (1.3) and (1.7) for each iteration after the first before using (1.4), while in Table 4 we applied only one of these two sets of constraints. However, applying only one of (1.3), (1.7) for each iteration is only one of the keys to the solution. In general, we must be careful which one of (1.3), (1.7) we apply. To illustrate, consider the iterative allocation in Table 6 for the same example considered in Tables 3 and 4. Here for iterations 2 and 3 we used only (1.7) and for iterations 4 and 5 only (1.3). The first three iterations are identical to those in Table 2 and hence are omitted. In this table (1.8) is violated for the final allocation for strata 3 and 5-9. Even more interesting would be a slight modification of Table 6 for which M_{10} is reduced to 17 with no other changes. If iterations 1-5 remain the same, there would now be an iteration 6 for which n_{10} is reduced from 18 to 17 and hence the final allocation would not satisfy

$$\sum_{h=1}^{H} n_h = n \quad (1.10)$$

In the next section we demonstrate how a specific iterative algorithm produces a final sample and a final value r that satisfies (1.2), (1.5), (1.6), (1.8) (1.9), and (1.10). In order for (1.2), (1.6), and (1.10) to be satisfied simultaneously it is clearly necessary that.

$$\sum_{h=1}^{H} m_h \leq n \leq \sum_{h=1}^{H} M_h \quad (1.11)$$

This is also sufficient. The general idea of the algorithm is that at each iteration either (1.3) or (1.7) is used but not both. Furthermore, if $n_h - M_h$ summed over those h violating (1.2) is greater than or equal to $m_h - n_h$ summed over those h violating (1.6), then (1.3) is used; otherwise (1.7) is used. More details are provided in the next section.

The algorithm described was recently applied to the sample allocation for the integrated National Compensation Survey program conducted by the Bureau of Labor Statistics. This application is described in detail in Ernst et al. (2002).

2. The Main Algorithm

We first introduce some additional notation. For the most part the notation will follow the notation of the previous section, with modifications to indicate the number of the iteration.

Let n_{hk}, $h = 1,...,H$, denote the number of sample units allocated to stratum h for iteration k. Let S_k, s_k denote the set of strata h for which the sample size has been fixed to be M_h, m_h, respectively for iteration k, and let

$$R_k = \{1,...,H\} - (S_k \cup s_k) \tag{2.1}$$

that is the set of the remaining strata. Note that, in particular, n_{h1} is the initial, directly proportional to aggregate MOS allocation and S_1, s_1 are prior to fixing the sample size of any strata; that is, $S_1 = s_1 = \varnothing$, $R_1 = \{1,...,H\}$. For each *k*, the strata in R_k are to have a common ratio, denoted r_k, for n_{hk} / T_h, and consequently we must have

$$r_k = \frac{n - \left(\sum_{h \in S_k} M_h + \sum_{h \in s_k} m_h \right)}{\sum_{h \in R_k} T_h} \tag{2.2}$$

$$\begin{aligned} n_{hk} &= M_h \text{ if } h \in S_k \\ &= m_h \text{ if } h \in s_k \\ &= r_k T_h \text{ if } h \in R_k \end{aligned} \tag{2.3}$$

It now remains to show the following. We first explain how S_k, s_k are obtained recursively for $k \geq 2$ in terms of S_{k-1}, s_{k-1} and $n_{h(k-1)}$, $h = 1,...,H$. This is key to the algorithm since (2.2) and (2.3) are defined in terms of S_k, s_k. Then we establish that there exists a smallest integer *K* for which both

$$S_K = S_{K-1}, \; s_K = s_{K-1} \tag{2.4}$$

and hence $n_{hK} = n_{h(K-1)}$ for all *h*. Then we first prove that the set of n_h and *r* defined by

$$n_h = n_{hK} = n_{h(K-1),} \;\; h = 1,...,H, \text{ and } r = r_K = r_{K-1} \tag{2.5}$$

satisfy (1.2) and (1.6); next that this set of n_h satisfies (1.10); and finally that the n_h and *r* satisfy (1.5), (1.8) and (1.9).

To recursively define S_k, s_k for $k \geq 2$, let

$$D_{k-1} = \sum_{h \in R_{k-1}} \max\{n_{h(k-1)} - M_h, 0\}, \tag{2.6}$$

$$d_{k-1} = \sum_{h \in R_{k-1}} \max\{m_h - n_{h(k-1)}, 0\} \tag{2.7}$$

$$\begin{aligned} S_k &= S_{k-1} \cup \{h : n_{h(k-1)} > M_h\} \text{ if } D_{k-1} \geq d_{k-1} \\ &= S_{k-1} \text{ if } D_{k-1} < d_{k-1} \end{aligned} \tag{2.8}$$

$$\begin{aligned} s_k &= s_{k-1} \cup \{h : n_{h(k-1)} < m_h\} \text{ if } d_{k-1} > D_{k-1} \\ &= s_{k-1} \text{ if } d_{k-1} \leq D_{k-1} \end{aligned} \tag{2.9}$$

The calculations of (2.6), (2.7) for the example of Table 4 are given in Table 5. To illustrate its use, since $D_1 \geq d_1$ we have by (2.8), (2.9) that $S_2 = \{1\}$, $s_2 = \varnothing$, from which, by (2.2), (2.3), the second iteration in Table 4 is obtained. This is equivalent to applying (1.3), (1.4) to the initial allocation.

To establish that there exists an integer *K* for which (2.4) holds, observe that $S_k \supset S_{k-1}$, $s_k \supset s_{k-1}$ for each $k \geq 2$, and consequently $R_k \subset R_{k-1}$ by (2.1). It follows from this last relation and the fact that $R_1 = \{1,...,H\}$, that either $R_k = R_{k-1}$ for some $k = 1,...,H+1$ or else $R_{H+2} = R_{H+1} = \varnothing$. Consequently, there is a smallest integer $K \leq H + 2$ such that $R_K = R_{K-1}$ and (2.4) holds for this *K*.

It follows from (2.2)-(2.4), (2.6)-(2.9) that the set of n_h, $h = 1,...,H$, defined by (2.5) satisfies (1.2), (1.6).

To show that this set of n_h satisfies (1.10), observe that unless $R_{K-1} = \varnothing$, (1.10) is satisfied by (2.2), (2.3) with $k = K - 1$, and (2.4), (2.5). However, we will show that $R_{K-1} \neq \varnothing$ by proving that

$$n_{(K-2)h} \leq M_h \text{ for some } h \in R_{K-2} \tag{2.10}$$

and

$$n_{(K-2)h} \geq m_h \text{ for some } h \in R_{K-2} \tag{2.11}$$

since (2.10), (2.11) combined with (2.1), (2.6)-(2.9) establishes that $R_{K-1} \neq \varnothing$. This is because if there is some *h* satisfying both (2.10), (2.11), then $h \in R_{K-1}$ for this *h*; while if there is a pair of strata, one satisfying (2.10) and the other (2.11), then one of these strata must be in R_{K-1} by (2.1), (2.6)-(2.9).

We will establish (2.10) by proving that for $k = 2,...,K-2$

$$\text{if } \sum_{h \in R_{k-1}} n_{h(k-1)} \leq \sum_{h \in R_{k-1}} M_h \text{ then } \sum_{h \in R_k} n_{hk} \leq \sum_{h \in R_k} M_h \tag{2.12}$$

Then since by (1.11) it follows that

$$n = \sum_{h \in R_1} n_{h1} \leq \sum_{h \in R_1} M_h \tag{2.13}$$

we combine (2.12), (2.13) to obtain by induction that

$$\sum_{h \in R_k} n_{hk} \leq \sum_{h \in R_k} M_h, \;\; k = 1,...,K-2 \tag{2.14}$$

and hence that (2.10) holds since $R_{K-2} \neq \varnothing$. The proof that (2.11) holds, which is omitted, is analogous.

To establish (2.12) we consider two cases, first $S_k \neq S_{k-1}$ and then $s_k \neq s_{k-1}$. In the former case it can be shown that

$$\begin{aligned}\sum_{h\in R_k} n_{hk} &= \sum_{h\in R_k} n_{h(k-1)} + D_{k-1} \\ &= \sum_{h\in R_{(k-1)}} n_{h(k-1)} - \sum_{h\in (R_{(k-1)}-R_k)} M_h \le \sum_{h\in R_k} M_h\end{aligned} \tag{2.15}$$

and in the latter case that

$$\begin{aligned}\sum_{h\in R_k} n_{hk} &= \sum_{h\in R_k} n_{h(k-1)} - d_{k-1} \\ &\le \sum_{h\in R_k} M_h + D_{k-1} - d_{k-1} \le \sum_{h\in R_k} M_h\end{aligned} \tag{2.16}$$

and hence (2.12) holds in both cases. Observe that the first relation of the chain (2.15) follows from (2.1)-(2.3), (2.6)-(2.9); the second from (2.1), (2.6)-(2.9); and the last relation from the hypothesis of (2.12). The first relation of (2.16) follows from (2.1)-(2.3), (2.6)-(2.9); the second from (2.1), (2.6)-(2.9); and the last relation from (2.9).

Finally, we will show that n_h, $h=1,...,H$, and r defined by (2.5) satisfies (1.5), (1.8), (1.9) by proving that for all $k=2,...,K$,

$$\text{if } n_{hj} \le r_j T_h \text{ for all } h\in S_j,\quad j=1,...,k-1,$$
$$\text{then } n_{hk} \le r_k T_h \quad \text{for all } h\in S_k \tag{2.17}$$
$$\text{if } n_{hj} \ge r_j T_h \text{ for all } h\in s_j,\quad j=1,...,k-1,$$
$$\text{then } n_{hk} \ge r_k T_h \quad \text{for all } h\in s_k \tag{2.18}$$

Since $S_1 = s_1 = \varnothing$, it is vacuously true that $n_{h1} \le r_1 T_h$ for all $h\in S_1$, $n_{h1} \ge r_1 T_h$ for all $h\in s_1$. Consequently, once (2.17), (2.18) are established, it follows by induction that

$$n_{hK} \le r_K T_h \quad \text{for all } h\in S_K \tag{2.19}$$
$$n_{hK} \ge r_K T_h \quad \text{for all } h\in s_K \tag{2.20}$$

Finally, (2.3), (2.5), (2.19), (2.20) establish (1.5), (1.8), (1.9).

Thus we need only establish (2.17), (2.18). We will only prove (2.17) since the proof of (2.18) is similar. To show (2.17) we let *g* denote the largest integer satisfying

$$g \le k \text{ and } S_{g-1} \ne S_k \tag{2.21}$$

If there is no *g* satisfying (2.21) then $S_k = S_1 = \varnothing$ and (2.17) is vacuously true. We will otherwise prove that

$$r_k \ge r_{g-1} \tag{2.22}$$

which establishes (2.17) since if $h\in S_{g-1}$ then

$$n_{hk} = n_{h(g-1)} \le r_{g-1}T_h \le r_k T_h \tag{2.23}$$

while if $h\in S_k - S_{g-1} = S_g - S_{g-1} \subset R_{g-1}$ then

$$n_{hk} = n_{hg} = M_h \le n_{h(g-1)} = r_{g-1}T_h \le r_k T_h \tag{2.24}$$

Note that the first relation in the chain (2.23) follows from (2.3) and $S_{g-1} \subset S_k$, and the second relation by the hypothesis of (2.17). The first two relations of (2.24) follow from (2.3) and $S_k = S_g$, and the third relation from (2.8). The fourth relation of (2.24) follows from (2.3) and $h\in R_{g-1}$.

To establish (2.22) we need only show that

$$\begin{aligned}\sum_{h\in R_g} n_{hk} = \sum_{h\in R_g} n_{hg} &= D_{g-1} + \sum_{h\in R_g} n_{h(g-1)} \\ &\ge d_{g-1} + \sum_{h\in R_g} n_{h(g-1)}\end{aligned} \tag{2.25}$$

and

$$\begin{aligned}\sum_{h\in R_g - R_k} (n_{hk} - n_{h(g-1)}) &= \sum_{h\in R_g - R_k} (m_h - n_{h(g-1)}) \\ &\le \sum_{h\in R_{g-1}} \max\{m_h - n_{h(g-1)}, 0\} = d_{g-1}\end{aligned} \tag{2.26}$$

since it follows from (2.3), (2.25), (2.26), that

$$r_k \sum_{h\in R_k} T_h = \sum_{h\in R_k} n_{hk} \ge \sum_{h\in R_k} n_{h(g-1)} = r_{g-1} \sum_{h\in R_k} T_h \tag{2.27}$$

To obtain (2.25) note that the first relation in (2.25) holds by combining

$$\sum_{h=1}^{H} n_{hk} = \sum_{h=1}^{H} n_{hg} = n \tag{2.28}$$

which follows from (2.2), (2.3), with the fact that $n_{hk} = n_{hg}$ for all $h \notin R_g$, which follows from (2.3), (2.8), (2.9). The second relation follows from (2.1)-(2.3), (2.6)-(2.9), (2.21). The final relation follows since $D_{g-1} \ge d_{g-1}$ by (2.8), (2.21).

To obtain (2.26), note that the first relation follows from (2.3) and the fact that $R_g - R_k \subset s_k$ by (2.21); the second relation from $R_g - R_k \subset R_{g-1}$; and the final relation from (2.7).

3. References

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Table 1. Example of Allocation with Constraints on Maximum Sample Sizes

Stratum	T_h	M_h	Iteration				Integer alloc.	n_h / T_h
			1	2	3	4		
1	85000	9	40.91	**9**	**9**	**9**	**9**	**0.0001**
2	19000	10	9.14	18.53	**10**	**10**	**10**	**0.0005**
3	9700	11	4.67	9.46	11.27	**11**	**11**	**0.0011**
4	6700	7	3.22	6.53	7.79	**7**	**7**	**0.0010**
5	3900	4	1.88	3.80	4.53	**4**	**4**	**0.0010**
6	2500	19	1.20	2.44	2.91	3.06	3	0.0012
7	2300	8	1.11	2.24	2.67	2.82	3	0.0012
8	5200	10	2.50	5.07	6.04	6.37	6	0.0012
9	8800	15	4.24	8.58	10.23	10.78	11	0.0012
10	6500	20	3.13	6.34	7.55	7.96	8	0.0012
Total	149600	113	72	72	72	72	72	

Table 2. Example of Allocation with Constraints on Minimum Sample Sizes

Stratum	T_h	M_h	m_h	Iteration			Integer alloc.	n_h / T_h
				1	2	3		
1	85000	100	1	40.91	32.38	31.91	32	0.00038
2	19000	100	1	9.14	7.24	7.13	7	0.00038
3	9700	100	7	4.67	*7*	*7*	*7*	*0.00072*
4	6700	100	1	3.22	2.55	2.52	3	0.00038
5	3900	100	2	1.88	*2*	*2*	*2*	*0.00051*
6	2500	100	6	1.20	*6*	*6*	*6*	*0.00240*
7	2300	100	3	1.11	*3*	*3*	*3*	*0.00130*
8	5200	100	6	2.50	*6*	*6*	*6*	*0.00115*
9	8800	100	4	4.24	3.35	*4*	*4*	*0.00045*
10	6500	100	1	3.13	2.48	2.44	2	0.00038
Total	149600	1000	32	72	72	72	72	

Table 3. Nonoptimal Allocation for Example with Both Sets of Constraints

Stratum	T_h	M_h	m_h	Iteration				Integer alloc.	n_h / T_h
				1	2	3	4		
1	85000	9	1	40.91	**9**	**9**	**9**	**9**	**0.0001**
2	19000	10	1	9.14	18.07	**10**	**10**	**10**	**0.0005**
3	9700	11	7	4.67	*7*	*7*	*7*	*7*	*0.0007*
4	6700	7	1	3.22	6.37	8.83	**7**	**7**	**0.0010**
5	3900	4	2	1.88	*2*	*2*	*2*	*2*	*0.0005*
6	2500	19	6	1.20	*6*	*6*	*6*	*6*	*0.0024*
7	2300	8	3	1.11	*3*	*3*	*3*	*3*	*0.0013*
8	5200	10	6	2.50	*6*	*6*	*6*	6	*0.0012*
9	8800	15	4	4.24	8.37	11.60	12.65	13	0.0014
10	6500	20	1	3.13	6.18	8.57	9.35	9	0.0014
Total	149600	113	32	72	72	72	72	72	

Table 4. Optimal Allocation for Example of Table 3

Stratum	T_h	M_h	m_h	Iteration						Integer alloc.	n_h/T_h
				1	2	3	4	5	6		
1	85000	9	1	40.91	**9**	**9**	**9**	**9**	**9**	**9**	**0.0001**
2	19000	10	1	9.14	18.53	**10**	**10**	**10**	**10**	**10**	**0.0005**
3	9700	11	7	4.67	9.46	11.27	10.46	10.60	10.48	10	0.0011
4	6700	7	1	3.22	6.53	7.79	7.23	**7**	**7**	**7**	**0.0010**
5	3900	4	2	1.88	3.80	4.53	4.21	**4**	**4**	**4**	**0.0010**
6	2500	19	6	1.20	2.44	2.91	*6*	*6*	*6*	*6*	*0.0024*
7	2300	8	3	1.11	2.24	2.67	*3*	*3*	*3*	*3*	*0.0013*
8	5200	10	6	2.5	5.07	6.04	5.61	5.68	*6*	*6*	*0.0012*
9	8800	15	4	4.24	8.58	10.23	9.49	9.62	9.50	10	0.0011
10	6500	20	1	3.13	6.34	7.55	7.01	7.10	7.02	7	0.0011
Total	149600	113	32	72	72	72	72	72	72	72	

Table 5. Contribution of Each Stratum to Value of D_k, d_k for Example of Table 4

Stratum	D_1	d_1	D_2	d_2	D_3	d_3	D_4	d_4	D_5	d_5
1	31.91	0								
2	0	0	8.53	0						
3	0	2.33	0	0	0.27	0	0	0	0	0
4	0	0	0	0	0.79	0	0.23	0		
5	0	0.12	0	0	0.53	0	0.21	0		
6	0	4.80	0	3.56	0	3.09				
7	0	1.89	0	0.76	0	0.33				
8	0	3.50	0	0.93	0	0	0	0.39	0	0.32
9	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0
Total	31.91	12.64	8.53	5.25	1.59	3.42	0.44	0.39	0	0.32

Table 6. Another Nonoptimal Allocation

Stratum	T_h	M_h	m_h	Iteration		n_h/T_h
				4	5	
1	85000	9	1	**9**	**9**	**0.0001**
2	19000	10	1	20.65	**10**	**0.0005**
3	9700	11	7	*7*	*7*	*0.0007*
4	6700	7	1	7.28	**7**	**0.0010**
5	3900	4	2	*2*	*2*	*0.0005*
6	2500	19	6	*6*	*6*	*0.0024*
7	2300	8	3	*3*	*3*	*0.0013*
8	5200	10	6	*6*	*6*	*0.0012*
9	8800	15	4	*4*	*4*	*0.0005*
10	6500	20	1	7.07	18	0.0028
Total	149600	113	32	72	72	