Properties of Callback Procedures Under Moderate Deviations From Specified Models October 2008

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Abstract: For many household and establishment surveys, initial contact and interview attempts by themselves do not produce satisfactory response rates. This often leads data-collection organizations to use various callback procedures to collect data from sample units that initially were nonrespondents. Analyses of the resulting data depend implicitly or explicitly on models for the response mechanism, conditional on the specific callback procedure. These models generally are acknowledged to be, at best, approximations to more complex underlying response processes. Consequently, it is important to explore the extent to which the properties of the callback-adjusted estimators may be sensitive to deviations from the assumed models. This paper develops a framework for exploring the impact of moderate deviations from assumed conditions, and presents some related simulation results.

Key words: Incomplete data; Logistic regression; Nonresponse adjustment; Simulation study.

1. Introduction

There is substantial literature on methods for estimation and inference from survey data subject to nonresponse. Some of this literature considers methods that account explicitly for the number of callbacks required to obtain the survey responses, (See, for example, Drew and Fuller (1980), Drew and Ray (1984), Potthoff et al. (1993) and references cited therein.) This paper provides some evaluations of the numerical performance of some specific callback-based methods proposed by Drew and Fuller (1980), and extensions thereof.

Specifically, Drew and Fuller (1980) consider a simple random sample of size n selected with replacement from a population of size N. Each sample unit is asked to provide survey responses; based on these responses, the unit would be placed into exactly one of *K* disjoint subpopulations.

Within the overall population, $(1-\gamma)100\%$ of the units are "hard core nonrespondents," i.e., regardless of any efforts by the survey organization, these units will not provide survey responses. For a unit that is in subpopulation k and is not a hard-core nonrespondent, the survey organization has probability q_k of obtaining a response on any given call attempt. Note that the per-call probabilities q_k are constant across call attempts *r=1....R*. Furthermore, the survey organization will make up to R call attempts, and will not stop those attempts until it obtains a response or it completes the R -th attempt.

Thus, if one does not collect any data from respondents except for their (non)response status and for respondents, their population classification, $k = 1,\dots,K$, the observed data are

n_0 = the number of units that did not respond by the $R-th$ call attempt ; and
n_{rk} = the number of units that responded on the $r-th$ attempt and that were subsequently classified into the $k-th$ subpopulation.

Drew and Fuller (1980, p. 639) then demonstrate that the vector $(n_0, n_{11},\dots n_{RK})$ follows a multinomial distribution with a sample size of $n = n_0 + \sum_{r=1}^{R}\sum_{k=1}^{K} n_{rk}$ and probabilities $(\pi_0, \pi_{\cdot 1},\dots\pi_{RK})$ where $\pi_0 = (1-\gamma) + \gamma\sum_{k=1}^{K}(1-q_k)^{r-1} q_k f_k$ and f_k equals the population proportion in category k.

For our current discussion, we will treat the terms f_k as known (e.g, from a recent population census or administrative record source) and focus on estimation of the (non)response-process parameters $\gamma, q_1, q_2, q_3, q_4$. For such cases, arguments in Drew and Fuller (1980, p. 639) indicate that the log-likelihood function for the aforementioned multinomial is

$$n_0 \ln(\pi_0) + \sum_{r=1}^{R} \sum_{k=1}^{K} \{n_{RK} \ln(\pi_{RK})\} \qquad (1.1)$$

where $\pi_{rk} = \gamma \left(1-q_k\right)^{r-1} q_k f_k$ and $\pi_0 = (1-\gamma) + \sum_{k=1}^{K} \left[\left(1-q_k\right)^{R-1}\right] f_k$. Algebraic simplification indicates that expression (1.1) equals

$$n_0 \ln\left\{(1-\gamma) + \gamma \sum_{k=1}^{K} \left(1-q_k\right)^2 f_k\right\} + \sum_{k=1}^{K} \left\{n_{1k} \ln\left(f_k \gamma_{qk}\right) + n_{2k} \ln\left(f_k \gamma_{qk}\left(1-q_k\right)\right)\right\} \qquad (1.2)$$

In the remainder of this paper we consider performance of maximum-likelihood point estimators based on the maximization of expression (1.1). Section 2 outlines the primary steps followed in this simulation study, reports results for sample sizes that are relatively large, i.e., n=250, 500, and 1000, and reports results for more modest sample sizes, n=50 and 100. Section 3 presents extensions of Sections 2 to cases involving misspecified logistic regression models. Section 4 reviews the main conclusions obtained from this simulation work and highlights some areas for future research.

2. Simulation-Based Evalulation of Estimators of $(\gamma, q_1, q_2, q_3, q_4)$

2.1 Design of the Simulation Study

For this section, we restricted attention to the case in which $K = 4$, $R = 2$ and $f_1 = f_2 = f_3 = f_4 = 0.25$. Thus, there are five parameters, γ, q_1, q_2, q_3, and q_4 that are to be estimated, each contained in the interval $(0,1)$. We study cases with n=50, 100, 250, 500, and 1000, with 1000 replicates in each case.

For this study, we set each of the true parameter values γ, q_1, q_2, q_3 and q_4 equal to 0.8. Using PROC IML, a SAS dataset with 1000 replicates is first generated. Each observation contains a count for the nine categories $(n_0, n_{11}, \ldots n_{24})$ derived from the generation of a multinomial random vector based on the assigned parameter values.

We used the SAS procedure PROC NLP to maximize the log-likelihood expression (1.2) with respect to the unknown parameters γ, q_1, q_2, q_3 and q_4 given the known population fractions f_1 through f_4 and the observed counts $n_0, n_{11}, \ldots n_{24}$. With the exceptions noted below, we initialize iterations for PROC NLP parameter values equal to the idealized values 0.8. We carried out related simulation runs with initial parameter values randomly selected from the interval $(0,1)$, but results are not detailed here.

2.2 Numerical Results for Large Sample Sizes

We studied a number of different Non-Linear Programming (NLP) techniques. For large samples with n>=250, the standard large-sample properties were generally satisfied, both for point estimators and for confidence interval estimators.

The numerical maximization techniques included the Newton-Raphson, the Newton-Raphson with Ridging, the Quasi-Newton, and the Conjugate-Gradient optimization techniques. In addition, we explored six different variance-estimation options included in PROC NLP.. Finally, we considered two different methods (Profile Likelihood and WALD) for construction of confidence intervals. For detailed discussion of these computational methods, see sections 28, 38, and 39 of SAS Institute (2002-2005).

2.3. Performance of Point Estimation Methods With Moderate Sample Sizes

Subsection 2.2 reported results for relatively large sample sizes: n=250, 500, and 1000. We also studied the properties of the same estimation procedures with n=50 and 100, and encountered the following numerical issues. First, when we allowed PROC NLP to initialize from a randomly selected starting point $\left(\gamma^{(0)}, q_1^{(0)}, q_2^{(0)}, q_3^{(0)}, q_4^{(0)}\right) \in [0,1]^5$, the resulting computed estimators varied widely, depending on the initial value. Second, for confidence interval computations we were unable to obtain confidence intervals with two of the techniques, and the two that did produce them produced intervals that were very wide. Third, we examined the numerical iterations obtained for one particular replicate for $n = 100$. This particular replicate number 558 was chosen because even when the idealized starting values of γ, q_1, q_2, q_3 and q_4 equal to 0.8 were used, the resulting computed estimates of q_3 and q_4 (0.9999 and 0.6123), respectively, were far from the true values equal to 0.8.

Table 1 presents each of the 32 iterations produced by PROC NLP for replicate 558 with initial values of 0.8 for each parameter. Note especially from Table 1 that the numerical values of the point estimates and the objective function are almost constant for iterates 9 through 27; and that the values for q_1, q_2, and q_4 change substantially between iterates 27 and 32. This numerical phenomenon is not uncommon in optimization. See e.g., Kennedy and Gentle (1980), p. 437.

Figure 1 displays the likelihood surface for replicate 558, with $\hat{\gamma} = 0.7669$, $\hat{q}_1 = 0.8259$, and $\hat{q}_3 = 0.999999983$ (equal to the numerical values for these parameters provided in the final row of Table 1) and with (q_2, q_4) allowed to vary over the rectangle [0.80, 0.90] x [0.60, 0.90] that contains the final point estimates $(\hat{q}_2, \hat{q}_4) = (0.890470816, 0.61231573)$. The likelihood values (displayed on the vertical axis and labeled “log-likelihood”) do not follow a simple quadratic-surface pattern generally used for large-sample approximations in likelihood-based point estimation and inference theory. Instead Figure 1 displays a local maximum near the point $(q_2, q_4) = (0.854, 0.687)$, a ridge that includes the abovementioned point, and a global maximum for the likelihood with q_2 and q_4 having values greater than 0.8.

In addition, Figure 2 displays the likelihood surface in a neighborhood of the final point estimates $(\hat{q}_2, \hat{q}_4) = (0.890470816, 0.61231573)$ with (γ, q_1, q_3) fixed at the final point estimates (0.7669314, 0.825864053, 0.999999983). Comparisons of Figures 1 and 2 indicate that the local maximum of the likelihood function in Figure 2 is substantially smaller than the local and global maxima displayed in Figure 1. Thus, when applied to data from replicate 558, with idealized initial values $(\gamma, q_1, q_2, q_3, q_4) = (0.8, 0.8, 0.8, 0.8, 0.8)$ the PROC NLP algorithm converges to the relatively small local maximum displayed in Figure 2, rather than the larger local maximum or the global maximum beyond the boundaries of Figure 2.

3. Performance Under Mis-Specified Logistic Regression Models

3.1 Description of Models and Simulation Methods

Section 2 presented results for estimation of the parameter vector $(\gamma, q_1, q_2, q_3, q_4)$ with the subpopulation proportions f_k known and the subpopulation-level response probabilities q_k treated as constants.

However, the callback literature uses more elaborate models that depend on the relationship between the probability of response and the observed auxiliary variables. This literature acknowledges that these are only approximations to the true probability model. For the remainder of the paper, we study the extent to which some estimation methods may be sensitive to differences between the true model and the model we actually use for estimation.

In our earlier simulation work, we used a simple version of the Drew and Fuller model in which the per-call response probability q_k depended only on the subpopulation membership of our sample unit.

Drew and Fuller (1980) also consider cases in which the q terms vary within subpopulations as quadratic functions of a predictor variable X:

$$q(X) = \beta_0 + \beta_1 X + \beta_2 X^2 \tag{3.1}$$

For the current section, we replaced the Drew-Fuller quadratic regression model (3.1) with a quadratic logistic regression model

$$\log[q(X)/\{1 - q(X)\}] = \beta_0 + \beta_1 X + \beta_2 X^2 \tag{3.2}$$

where β_0, β_1 and β_2 are fixed coefficients and X is a known predictor variable. As in Section 2, we used $\gamma = 0.8$, and $R=2$. In addition, we used X generated from a normal distribution with mean equal to 0 and standard deviation equal to 0.5. We then partitioned the population into $K=8$ groups. Group member ship was determined by whether a given unit had its X value contained in the intervals $[-\infty, x_{0.125}]$, $[x_{0.25}, x_{0.375}]$, $(x_{0.375}, x_{0.5}]$, $(x_{0.5}, x_{0.625}]$, $(x_{0.625}, x_{0.75}]$, $(x_{0.75}, x_{0.875}]$, $(x_{0.875}, \infty)$, respectively, where x_q is defined to equal the q-th quantile of the $N(0,0.25)$ distribution. Thus, $K=8$ and $f_k = 0.125$ for each $k=1,\ldots,8$.

Also, in a logistic regression extension of Drew and Fuller (1980, p. 639) , we defined $q_k = \exp(L_k)/\{1+\exp(L_k)\}$ where $L_k = \beta_0 + \beta_1 A_k + \beta_2 A_k^{\,2}$ and A_k equals the $(k-0.5)/K$ quantile of the $N(0,0.25)$ distribution.

Based on model (1.1) with value q_k determined by expression (3.2) and specified coefficient vectors $(\beta_0, \beta_1, \beta_2)$ equal to $(0,0.25,0.01)$, $(0,0.25,0.10)$ or $(0,0.25,0.50)$, we computed the resulting probability vectors $\pi_\beta = (\pi_{\beta 0}, \pi_{\beta 11}, \ldots \pi_{\beta 28})$. For each of the sample sizes $n=250, 500, 1000$, we generated 1000 independent random vectors $(n_0, n_{11},\ldots, n_{28})$ according to a multinomial (n, π_β) distribution.

For each generated random vector $(n_0, n_{11}, \ldots n_{28})$, we then fit two models: the correctly specified quadratic logistic model (3.2) and the incorrectly specified linear logistic model

$$\log[q(X)/\{1 - q(X)\}] = \beta_0 + \beta_1 X \tag{3.3}$$

In all cases, we used PROC NLP to compute the resulting vector of point estimators $(\hat{\gamma}_A, \hat{\beta}_{0A}, \hat{\beta}_{1A}, \hat{\beta}_{2A})$ for model (1.1)-(3.2) and $(\hat{\gamma}_B, \hat{\beta}_{0B}, \hat{\beta}_{1B})$ for model (1.1)-(3.3) .

3.2 Numerical Results for Large Sample Sizes

Table 2 displays results for the mean, standard deviation, mean squared error and quantiles of $\hat{\gamma}$ for each combination of n= 1000, and $\beta_2 = 0.01$, 0.10 and 0.50, and based on inclusion or exclusion, respectively, of β_2 from the fitted model. Note especially that except for the case in which $\beta_2 = 0.50$, the mean squared errors for $\hat{\gamma}$ are smaller (usually slightly smaller) for the misspecified model (1.1)-(3.3) relative to the correctly specified model (1.1)-(3.2). Thus, for the cases studied here, misspecification of the logistic regression model appears to have a relatively small impact on the performance of $\hat{\gamma}$, the estimator of the proportion of the population outside the hard-core nonresponse group. Qualitatively similar conclusions applied to numerical results for n=250 and 500, which are not presented here in detail.

To study further the effect of model misspecification, we computed the true response probabilities $p_k = \gamma \exp\{L_k\}/\{1+\exp(L_k)\}$ where $L_k = \beta_0 + \beta_1 A_k + \beta_2 A_k^{\,2}$; the estimated probabilities based on the correctly specified model fit $\hat{p}_{krA} = \exp(\hat{L}_{krA})/\{1+\exp(\hat{L}_{krA})\}$ where $\hat{L}_{krA} = \hat{\beta}_{0rA} + \hat{\beta}_{1rA}A_k + \hat{\beta}_{2rA}A_k^2$ and the estimated probabilities based on the incorrectly specified model fit $\hat{p}_{krB} = \exp(\hat{L}_{krB})/\{1+\exp(\hat{L}_{krB})\}$ where $\hat{L}_{krB} = \hat{\beta}_{0rB} + \hat{\beta}_{1rB}A_k$.

In the notation above, the subscript *r* refers to the *r-th* replication in our simulation study.

We then computed the differences $d_{krA} = \hat{p}_{krA} - p_k$ and $d_{krB} = \hat{p}_{krB} - p_k$ for each *k=1, 2, ...,8* and for each combination of *n=250, 500, 1000* and β_2 *=0.01, 0.10,* and *0.50*. Table 3 shows the resulting simulation based means $\bar{d}_{k.A} = (1000)^{-1}\sum_{r=1}^{1000} d_{krA}$ and $\bar{d}_{k.B} = (1000)^{-1}\sum_{r=1}^{1000} d_{krB}$ for the cases $\beta_2 =$ 0.01, 0.10, and 0.50 and for $k = 1,\ldots,8$. The corresponding true probabilities, p_k are also displayed in each table.

Note that the estimated biases $\bar{d}_{k.A}$ and $\bar{d}_{k.B}$ are small relative to p_k for $\beta_2 = 0.01$, and are still fairly small for $\beta_2 = 0.10$. However, for $\beta_2 =$ 0.50, the estimated biases $\bar{d}_{k.B}$ for the misspecified model case are no longer small relative to p_k, especially for the tail-quantile groups 1 and 8.

4. Discussion

4.1 Summary of Results

This paper has studied some properties of maximum-likelihood estimators arising from a nonresponse-callback model developed by Drew and Fuller (1980). Section 2 considered a relatively simple form of the model, with primary emphasis on the estimators $\hat{\gamma}$ (the proportion of the population not classified as "hard-core nonrespodents") and $\hat{q}_k$ (the per-call response probability for sample units in subpopulation *k*). For relatively large sample sizes, the point estimators $\hat{\gamma}$ and $\hat{q}_k$, and associated confidence interval estimators, had properties that were generally consistent with standard large-sample theory. For relatively small sample sizes, however (e.g. *n*=50 or 100), the likelihood surface was irregular in a substantial number of cases, leading to issues related to multiple local maxima and boundary-case global maxima.

Section 3 considered a more complex case, in which the units that were not "hard-core nonrespondents" had response probabilities that followed a logistic regression model with a link function that was a quadratic function of a single predictor *X*. The simulation work in Section 3 evaluated properties of point estimators for γ and for unit-level response probabilities under two models:

(a) the correctly specified quadratic-logistic model; and
(b) an incorrectly specified linear-logistic model

Comparison of results from cases (a) and (b) led to evaluation to the extent to which the estimators are sensitive to model misspecification.

4.2 Future Work

The simulations work presented here could be extended to several additional evaluations of properties of estimators under callback models that are properly specified or mis-specified, respectively. Prominent examples include callback-adjusted estimators of population means, totals and related nonlinear parameters. In addition, one may evaluate the properties of the same estimators under: (1) a complex sample design; and (2) a callback design that depends on auxiliary data for all sample units, and that may involve additional interventions like change in collection mode or follow-up by specialists in conversion of relevant respondents. For example, many household and establishment surveys use designs that include stratification, multistage sampling, and unequal probabilities of selection.

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Table 1: Iterations of PROC NLP with Initial Starting Points of 0.8

Iteration	γ	q_1	q_2	q_3	q_4	Log-likelihood
0	0.8	0.8	0.8	0.8	0.8	-187.7108716
1	0.798124342	0.800594735	0.800718235	0.82	0.8002368	-187.3299358
2	0.792967528	0.801841563	0.807140666	0.854088797	0.794811743	-186.5240792
3	0.781165147	0.803911994	0.821236984	0.913236762	0.778853691	-184.9718122
4	0.780004406	0.804185146	0.822911747	0.921913086	0.776838353	-184.758612
5	0.77668641	0.804684807	0.826667591	0.937910477	0.771714462	-184.3330267
6	0.77557638	0.804880021	0.828049032	0.944119429	0.769835902	-184.1721235
7	0.772475701	0.805218872	0.830979773	0.956074728	0.765519987	-183.8458697
8	0.762817178	0.806534544	0.841345368	0.999955231	0.750526227	-182.7008457
9	0.762817171	0.806534562	0.841345434	0.999959708	0.750526142	-182.7007507
10	0.762816874	0.806534886	0.841346908	0.999972477	0.750524205	-182.7004598
11	0.762815976	0.806535256	0.841349022	0.999983311	0.750521315	-182.7001967
12	0.762814337	0.8065356	0.841351363	0.999993749	0.750517974	-182.6999342
13	0.762814214	0.806535619	0.841351507	0.999994374	0.750517762	-182.6999183
14	0.762814082	0.806535638	0.841351653	0.999994997	0.750517548	-182.6999023
15	0.762813949	0.806535656	0.841351798	0.999995621	0.750517334	-182.6998864
16	0.762813681	0.806535694	0.841352089	0.999996868	0.750516905	-182.6998544
17	0.762813144	0.806535768	0.841352672	0.999999361	0.750516046	-182.6997905
18	0.762813131	0.80653577	0.841352687	0.999999425	0.750516024	-182.6997889
19	0.762813103	0.806535774	0.841352716	0.999999553	0.75051598	-182.6997856
20	0.762813048	0.806535782	0.841352776	0.999999808	0.750515892	-182.6997791
21	0.762813044	0.806535782	0.841352781	0.999999827	0.750515885	-182.6997786
22	0.762813036	0.806535783	0.84135279	0.999999866	0.750515872	-182.6997776
23	0.762813019	0.806535786	0.841352808	0.999999943	0.750515846	-182.6997756
24	0.762813018	0.806535786	0.841352809	0.999999948	0.750515844	-182.6997755
25	0.762813015	0.806535786	0.841352812	0.99999996	0.75051584	-182.6997752
26	0.76281301	0.806535787	0.841352817	0.999999983	0.750515832	-182.6997746
27	0.75725268	0.829074546	0.908111833	0.999999983	0.665848579	-182.1898916
28	0.765442893	0.8262262	0.8935267	0.999999983	0.616961793	-182.1092089
29	0.76691338	0.825870342	0.89056497	0.999999983	0.612353133	-182.1080021
30	0.7669314	0.825864053	0.890470816	0.999999983	0.61231573	-182.1080014
	0.7669314	0.825864053	0.890470816	0.999999983	0.61231573	-182.1080014

Table 2: Properties of $\hat{\gamma}$ Under Misspecified and Correctly Specified Models for Response Probabilities

True Value β_2	SampleSize n	$\beta_2$in estimated model	Mean	Standard Deviation	MSE=(Mean-.8)^2 +Variance	$q_{.05}$	$q_{.25}$	$q_{.50}$	$q_{.75}$	$q_{.95}$
0.01	1000	Y	.810673	.05009	.002624	.738633	.774378	.805883	.843706	.897679
0.01	1000	N	.808625	.04989	.002564	.736384	.773012	.803033	.840452	.895482
0.10	1000	Y	.810168	.04770	.002379	.740177	.777672	.803543	.839627	.896274
0.10	1000	N	.805726	.04755	.002294	.736436	.772796	.800243	.834305	.891577
0.50	1000	Y	.806844	.03387	.001197	.754706	.783063	.805138	.827515	.863406
0.50	1000	N	.782682	.03033	.001220	.736899	.761982	.781228	.801638	.838138

Table 3: Differences Between Estimated Model Probability and True Probability for β_2=0.01, 0.10, and 0.50 with n=1000

True Value β_2	Sample Size NT	β_2 in model?	Mean d_1	Mean d_2	Mean d_3	Mean d_4	Mean d_5	Mean d_6	Mean d_7	Mean d_8
0.01	1000	Y	-0.00034	-0.00048	-0.00031	-0.00022	-0.00020	-0.00022	-0.00034	-0.00162
0.01	1000	N	-0.00258	-0.00014	0.000769	0.001110	0.001052	0.000560	-0.00064	-0.00403
0.01		True P	0.328769	0.357379	0.376068	0.392185	0.407914	0.424884	0.44573	0.480307
0.10	1000	Y	-0.00080	0.00014	0.000448	0.00045	0.00024	-0.00019	-0.00098	-0.00394
0.10	1000	N	-0.02571	0.003131	0.012908	0.016263	0.015354	0.010071	-0.00199	-0.03320
0.10		True P	0.370419	0.371432	0.380355	0.392630	0.408359	0.429165	0.459650	0.519971
0.50	1000	Y	0.00030	-0.00032	-0.00092	-0.00101	-0.00074	-0.000004	0.001475	0.002417
0.50	1000	N	-0.13315	0.009448	0.060106	0.077939	0.074327	0.049064	-0.00757	-0.12682
0.50		True P	0.550820	0.434261	0.399451	0.394609	0.410338	0.448095	0.51932	0.661116

Figure 1: Surface Graph of the Likelihood Surface for Replicate 558

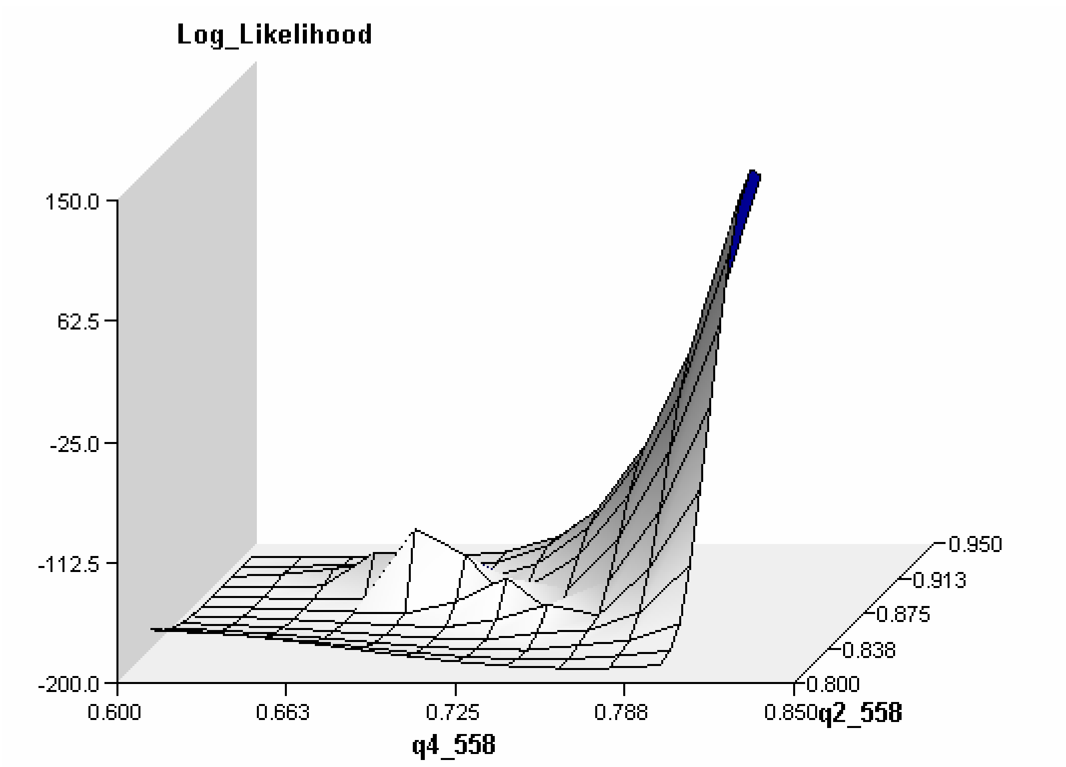

Figure 2: Likelihood Surface in the Neighborhood of the Final Point Estimates for Replicate 558