

Time Series Analysis of Price Indices

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Abstract

The International Price Program of the Bureau of Labor Statistics estimates monthly indices on the changes in import and export prices for merchandise and services. The data are collected through a complex sample of establishments using monthly reported data. Consequently, in time series analyses of IPP data, there is potential interest in the variances and autocorrelation functions of both sampling and measurement errors, as well as the underlying true price-index series. This paper presents several analytic methods that one may use to estimate the parameters of these terms.

Key Words: Auto-regressive moving average (ARMA) models, Bootstrap estimator, International Price Program.

1. Introduction

1.1 The U.S. International Price Program

The International Price Program (IPP) of the Bureau of Labor Statistics (BLS) produces two of the major price series for the United States: the Import Price indices and the Export Price indices. The IPP, as the primary source of data on price change in the foreign trade sector of the U.S. economy, publishes index estimates of price change for internationally traded goods using three primary classification systems - Harmonized System (HS), Bureau of Economic Analysis End Use (BEA), and North American Industry Classification System (NAICS). For general background on the IPP and its sample design, weight structure and index estimation, and on the prospective bootstrap variance estimators, see Chapter 15 of the Bureau of Labor Statistics Handbook of Methods, International Price indexes, Bobbitt et al.(2005, 2007), Chen et al.(2007), and Cho et al.(2007).

1.2 Time Series Models for Finite-Population Price Indices and Estimation Errors

Consider a set of finite-population price indices θ_{tg} defined for periods $t = 1, \dots, T$ and two-digit product groups called chapters $g = 1, \dots, G$. In addition, define the column vectors $\theta_{(T)g} = (\theta_{1g}, \dots, \theta_{Tg})'$. We will assume that these finite-population index vectors were generated through a superpopulation process ξ such that

$$E_{\xi}(\theta_{tg}|X_{tg}) = \mu_{tg} = \mu_{tg}(X_{tg}, \beta) \quad (1)$$

where X_{tg} is a $k \times 1$ vector of known predictors and β is an $r \times 1$ vector of parameters. In addition, define the finite-population deviation terms

$$a_{tg} = \theta_{tg} - \mu_{tg} \quad (2)$$

and the vectors $\mu_{(T)g} = (\mu_{1g}, \dots, \mu_{Tg})'$ and $a_{(T)g} = (a_{1g}, \dots, a_{Tg})'$. Also, we will assume that under the superpopulation model ξ , a_{tg} follows a stationary time series

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model, and we define the $T \times T$ covariance matrix

$$V_{(T)g} = V_{\xi} \left(a_{(T)g} \right). \quad (3)$$

Based on the sample design and estimation methods reviewed in Section 2.1 and Appendix A, we produce an estimator $\hat{\theta}_{tg}$ of the finite-population index θ_{tg} . We define the estimation errors

$$e_{tg} = \hat{\theta}_{tg} - \theta_{tg} \quad (4)$$

and the vectors $e_{(T)g} = (e_{1g}, \dots, e_{Tg})'$ and $\hat{\theta}_{(T)g} = (\hat{\theta}_{1g}, \dots, \hat{\theta}_{Tg})'$. We assume that the errors e_{tg} follow a stationary time series model with mean equal to zero, and we define the covariance matrix

$$V_{(T)eg} = V_{p\xi} \left(e_{(T)g} \right) \quad (5)$$

where $V_{p\xi}(\cdot)$ refers to evaluation of a covariance function with respect to both the sample design and the underlying superpopulation model. In some cases, it will be useful to decompose this covariance matrix as

$$V_{(T)eg} = V_{(T)Seg} + V_{(T)NSeg} \quad (6)$$

where $V_{(T)Seg}$ represents the covariance matrix that $e_{(T)g}$ would have if estimation error arose only from pure sampling error associated with the prescribed sample design, and $V_{(T)NSeg}$ error covariance matrix arising from the effects of nonsampling errors including, e.g., nonresponse, reporting error and imputation.

Under the conditions developed above, $(\theta'_{(T)g}, \hat{\theta}'_{(T)g})'$ has a mean vector $(\mu'_{(T)g}, \mu'_{(T)g})'$ and a covariance matrix

$$\begin{pmatrix} V_{(T)g} & V_{(T)g} \\ V_{(T)g} & V_{(T)g} + V_{(T)eg} \end{pmatrix}. \quad (7)$$

Assume for the moment that all of the mean and covariance parameters are known. Under the stated conditions, a linear least-squares predictor of $\theta_{(T)g}$ is

$$\theta^*_{(T)g} = \mu_{(T)g} + V_{(T)g} \left(V_{(T)g} + V_{(T)eg} \right)^{-1} \left(\hat{\theta}_{(T)g} - \mu_{(T)g} \right). \quad (8)$$

Specific versions of the predictor (8) have been considered by several authors within the context of time series analysis of sample survey estimators. See, e.g., Scott and Smith (1974), Scott, Smith and Jones (1977), Tiller (1992), Bell (1993, 1995), Pfeffermann (1991), Brunsdon and Smith (1998), and references cited therein. Under the stated conditions, the variance of the prediction error $\theta^*_{(T)g} - \theta_{(T)g}$ is

$$V_{(T)g} - V_{(T)g} \left(V_{(T)g} + V_{(T)eg} \right)^{-1} V_{(T)g} \quad (9)$$

while the variance of the error $\hat{\theta}_{(T)g} - \theta_{(T)g}$ is $V_{(T)eg}$.

1.3 Use of Time Series Methods to Improve Price Index Prediction and Inference

The remainder of this paper uses the decompositions (2) and (4), to develop approximations for the autocovariance functions of θ_{tg} , a_{tg} , and e_{tg} , and to suggest some

applications to estimation and inference for the underlying true price index series. Section 2 reviews some simple estimators of a fixed price-index series, predictors of a corresponding random series, and some related estimators of parameters for the underlying mean function. Section 3 considers several methods for estimation of the autocovariance structure of estimation errors e_{tg} . Specifically, Section 3.1 reviews cross-sectional bootstrap methods used to estimate sampling error variances for the IPP, and discusses extension of these methods to provide a direct estimator $\hat{V}_{(T)eg}$ of the $T \times T$ sampling-error variance-covariance matrix $V_{(T)eg}$. Section 3.2 outlines the use of Yule-Walker methods to estimate the parameters of an autoregressive moving average (ARMA) model to the matrix $\hat{V}_{(T)eg}$. Section 4 presents a set of methods to estimate the autocovariance structure of estimation errors based on a large number of replicates from a simulation study. Section 5 reviews the primary results of this paper and Section 6 suggests several areas for additional research.

2. Improved Prediction of the True Index Series

2.1 Estimation of the Fixed Finite Population Index $\theta_{(T)g}$

The IPP uses items that are initiated and re-priced every month to compute its indices of price change. These indices are calculated using a modified Laspeyres index formula. For each classification system, the IPP calculates the estimates of price change using an aggregation tree structure beginning with items, weight groups, classification groups, stratum-lower, stratum-upper, . . . , and finally overall. Weight groups are defined by the intersection of establishment and product classification group. Note that there could be many different levels, such as stratum-lower (which is right above the stratum-lower) and stratum-upper (which is above the stratum-lower). The formula is basically the same for all levels: each parent's index is computed from its children's indices. For example, a stratum index is computed from the stratum's children's indices. These children could be classification groups, stratum-lowers, stratum-uppers or any combination of them. Define $Child[h]$ to be the set of all stratum-lowers, stratum-uppers or classification groups directly below stratum h in an aggregation tree. In practice, θ_{th} , a short term ratio (STR) for a stratum h at time t , is computed from the weighted long term ratios, I_{tc} and I_{t-1c} , from its children's set.

$$\hat{\theta}_{th} = \sum_{c \in Child[h]} w_c I_{tc} \left(\sum_{c \in Child[h]} w_c I_{t-1c} \right)^{-1} \quad (10)$$

where w_c is the weight of an element c of $Child[h]$, and I_{tc} the long term price ratio of c at time t . $\hat{\theta}_{th}$ is then used in computing I_{th} , a long term ratio for a stratum h at time t .

$$I_{th} = \prod_{u=0}^t \hat{\theta}_{uh} \quad (11)$$

This general formula (10) and (11) are used until the desired aggregation level index is obtained.

2.2 Estimated Parameters of the Mean Structure

$\theta_{(T)g}$ can be modeled as a sum of mean and residual components, i.e.,

$$\theta_{(T)g} = T_{(T)g} + S_{(T)g} + I_{(T)g}$$

where the mean term

$$\mu_{tg} = T_{(T)g} + S_{(T)g}.$$

Under the conditions developed in section 1.2, one may consider estimation of the regression-model parameters β through the method of ordinary least squares, e.g., through minimization of

$$\sum_{t=1}^T \left\{ \hat{\theta}_{tg} - \mu_{tg}(X_{tg}, \beta) \right\}^2$$

with respect to β , for a known set of predictors X_{tg} and known functional form $\mu_{tg}(\cdot, \cdot)$. In addition, one could consider construction of generalized least squares estimators of μ_{tg} based on minimization of

$\left(\hat{\theta}_{(T)g} - \mu_{(T)g} \right)' \left(V_{(T)eg} + V_{(T)eg} \right)^{-1} \left(\hat{\theta}_{(T)g} - \mu_{(T)g} \right)$. Some simple examples of mean functions are

$$\mu_{tg} = \beta_0 + \beta_1 t + \beta_2 \sin(\beta_3 + \beta_4 t) \quad (12)$$

for time trends and cyclical patterns; or

$$\mu_{tg} = \beta_{\text{mod}_{12}[t]}$$

for month effects.

3. Use of Sample Data to Estimate the Autocovariance Structure of Estimation Errors

In analysis of time series data as considered in Section 2, it is often important to have estimators of the variances of the true series $\theta_{(T)g}$ (and components thereof) and the estimation error term $e_{(T)g}$. For example, estimation of $V_{(T)eg}$ allows computation of standard errors, for $\hat{\theta}_{(T)g}$ and related confidence intervals. This section reviews bootstrap estimators $\hat{V}_{eg \text{ Boot}}$ of $V_{(T)eg}$ computed only from cross-sectional variation in replicate-based estimators of $\theta_{(T)g}$, and considers alternative estimators of $V_{(T)eg}$ based on imposition of time series parametric structure.

3.1 Use of Bootstrap Methods to Estimate Sampling Error Variances and Covariances from Complex Sample Data

The bootstrap method for the iid case has been extensively studied since Efron proposed his bootstrap method in 1979. The original bootstrap method was then modified to handle complex issues in survey sampling, and results were extended to cases such as stratified multistage designs. Rao and Wu (1988) provided an extension to stratified multistage designs and their main technique to apply the bootstrap method to complex survey data was scaling. The estimate of each resampled cluster was properly scaled so that the resulting variance estimator reduced to the standard unbiased variance estimator in the linear case

$$\hat{V}_{eg \text{ Boot}} = (B - 1)^{-1} \sum_{b=1}^B \left(\hat{\theta}_{g(b)} - \hat{\theta}_{g(\cdot)} \right)^2. \quad (13)$$

The methods reviewed here extended readily to the T -dimensional vectors $\hat{\theta}_{(T)g} = \left(\hat{\theta}_{1g}, \dots, \hat{\theta}_{Tg} \right)'$ and lead to a bootstrap estimator of the $T \times T$ variance-covariance

matrix $\hat{V}_{(T)eg Boot}$ of the sampling errors e_{tg} defined in expression (4) with (i, j) th element

$$\hat{V}_{(T)eg Boot ij} = (B - 1)^{-1} \sum_{b=1}^B \left(\hat{\theta}_{ig(b)} - \hat{\theta}_{ig(\cdot)} \right) \left(\hat{\theta}_{jg(b)} - \hat{\theta}_{jg(\cdot)} \right)'. \quad (14)$$

In principle, we may compute an estimated covariance matrix $\hat{V}_{(T)eg Boot}$ for large values of T . However, as T increases, analyses of data from the IPP may encounter issues with weights that are missing or unreliable (thus making it difficult or impossible to compute $\hat{\theta}_{tg}$) or unreliable recorded values of item-level short term price ratios (STRs).

3.2 Improving Autocovariance Matrix Estimators through Use of Parsimonious Time Series Models

For moderate or large values of T , one may consider imposing some time series parametric structure on the covariance matrix $\hat{V}_{(T)eg}$. Under conditions, this may lead to an improved and more stable estimator, $V_{(T)eg}^*$, say. For example, suppose that our error terms e_{tg} follow the AR(1) model: $e_{tg} = \rho_g e_{t-1,g} + d_{tg}$ where $d_{tg} \stackrel{iid}{\sim} (0, \sigma_{dg}^2)$, $t = 1, \dots, T$. Then

$$V_{(T)eg} = \left(\frac{\sigma_{dg}^2}{1 - \rho_g^2} \right) \begin{pmatrix} 1 & \rho & \rho^2 & \dots & \rho^{T-1} \\ \rho & 1 & \rho & \dots & \rho^{T-2} \\ \rho^2 & \rho & 1 & \dots & \rho^{T-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho^{T-1} & \rho^{T-2} & \rho^{T-3} & \dots & 1 \end{pmatrix}. \quad (15)$$

Note especially that the general $T \times T$ covariance matrix $V_{(T)eg}$ has $T(T + 1)/2$ parameters to be estimated while under the AR(1) model the simplified covariance matrix (15) has only two parameters to be estimated: σ_{dg}^2 and ρ . For AR(1), we considered estimating (σ_{dg}^2, ρ_g) through a methods-of-moments fit of the bootstrap-based $T \times T$ matrix $V_{(T)eg}$ to (15). Define $(\hat{\sigma}_{dg}^2, \hat{\rho}_g)$ to be resulting vectors of parameter estimators for the AR(1). In keeping with standard approaches (e.g., Shumway and Stoffer, Section 3.6, 2006) Yule-Walker (YW) equations for AR(1) are given by

$$\gamma(1) = \rho_g \gamma(0) \quad (16)$$

$$\sigma_{dg}^2 = \gamma(0) - \rho_g \gamma(1) \quad (17)$$

where $\gamma(h) = cov(e_{t+h}, e_t)$. Similarly, AR(2) can be expressed

$$e_{tg} = \phi_{1g} e_{t-1,g} + \phi_{2g} e_{t-2,g} + d_{tg}.$$

Its YW equations are given by

$$\gamma(1) = \phi_{1g} \gamma(0)$$

$$\gamma(2) = \phi_{1g} \gamma(1) + \phi_{2g} \gamma(0)$$

$$\sigma_{dg}^2 = \gamma(0) - \phi_{1g} \gamma(1) - \phi_{2g} \gamma(2)$$

where $\gamma(0) = T^{-1} \sum_{j=1}^T \hat{V}_{(T)egjj}$ the mean of the diagonal entries of $\hat{V}_{(T)eg}$, and $\gamma(1) =$

$(T - 1)^{-1} \sum_{j=1}^{T-1} \hat{V}_{(T)egj,j+1}$ the mean of the first off-diagonal entries of $\hat{V}_{(T)eg}$.

For a general AR(p) model: $e_{tg} = \sum_{l=1}^p \phi_{gl}e_{t-l,g} + d_{tg}$, YW equations in matrix notation are given as

$$\mathbf{V}_p \phi_g = \gamma_p \quad (18)$$

$$\sigma_{dg}^2 = \gamma(0) - \phi_g' \gamma_p. \quad (19)$$

where \mathbf{V}_p is a $p \times p$ variance-covariance matrix whose k th off-diagonal entry C_k is given for $k = 0, \dots, p-1$:

$$C_k = (p-k)^{-1} \sum_{j=1}^{p-k} \hat{V}_{(T)egj,j+k}.$$

By solving (18) and (19), YW estimators are:

$$\hat{\phi}_g = \hat{V}_p^{-1} \hat{\gamma}_p \quad (20)$$

$$\hat{\sigma}_{dg}^2 = \hat{\gamma}(0) - \hat{\gamma}_p' \hat{V}_p^{-1} \hat{\gamma}_p \quad (21)$$

$$\hat{v}(\hat{\phi}_g) = n^{-1} \hat{V}_p^{-1} \sigma_{dg}^2. \quad (22)$$

Similarly under the MA(1) model: $e_{tg} = \alpha_g d_{t-1,g} + d_{tg}$

$$V_{(T)eg} = \sigma_{dg}^2 (1 + \alpha_g^2) \begin{pmatrix} 1 & \alpha_g & 0 & \cdots & 0 \\ \alpha_g & 1 & \alpha_g & \cdots & 0 \\ 0 & \alpha_g & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix} \quad (23)$$

which again involves only two parameters to be estimated. For MA(1) model,

$$\gamma(0) = (1 + \alpha_g^2) \sigma_{dg}^2$$

$$\gamma(1) = \alpha_g \sigma_{dg}^2$$

$$\rho(1) = \frac{\alpha_g}{(1 + \alpha_g^2)}.$$

Solutions of this pair of equations in two unknown will lead to the estimators

$$\hat{\alpha}_g = \frac{1 - \sqrt{1 - 4\hat{\rho}(1)^2}}{2\hat{\rho}(1)}$$

$$\hat{\sigma}_{dg}^2 = \frac{2\hat{\gamma}(0)\hat{\rho}(1)^2}{1 - \sqrt{1 - 4\hat{\rho}(1)^2}}.$$

Similarly, an MA(2) model can be expressed as

$$e_{tg} = d_{tg} + \alpha_{1g}d_{t-1,g} + \alpha_{2g}d_{t-2,g}$$

and its equations are:

$$\gamma(0) = (1 + \alpha_{1g}^2 + \alpha_{2g}^2) \sigma_{dg}^2$$

$$\gamma(1) = (\alpha_{1g} + \alpha_{1g}\alpha_{2g}) \sigma_{dg}^2$$

$$\gamma(2) = \alpha_{2g} \sigma_{dg}^2.$$

4. Use of Simulation Methods to Approximate the Autocovariance Structure of Estimation Errors

In some cases, one may consider supplementation of the bootstrap-based estimator $\hat{V}_{(T)eg}$ with variance estimators obtained from a simulation study.

Specifically, consider a simulation study that generates a single finite population U of size N , covering P periods. One then uses a given sample design D to select R samples $S_r, r = 1, \dots, R$ independently from U . Based on data from each sample S_r , one computes the P -dimensional vector of index estimators $\hat{\theta}_{(P)gr}, r = 1, \dots, R$. The resulting $P \times P$ matrix of second moments is

$$\tilde{V}_{(P)g} = (R - 1)^{-1} \sum_{r=1}^R \left(\hat{\theta}_{(P)gr} - \hat{\theta}_{(P)g\cdot} \right) \left(\hat{\theta}_{(P)gr} - \hat{\theta}_{(P)g\cdot} \right)' \quad (24)$$

where $\hat{\theta}_{(P)g\cdot} = R^{-1} \sum_{r=1}^R \hat{\theta}_{(P)gr}$. One may then consider two approaches to uses of $\tilde{V}_{(P)g}$ to estimate time series model parameters.

In parallel with the estimators developed in Section 3, one may compute estimators of ARMA(p, q) parameters based on a method-of-moments fit of $\tilde{V}_{(P)g}$. Call the resulting estimators $(\tilde{\sigma}_{dg}^2, \tilde{\rho}_{1g}, \dots, \tilde{\rho}_{pg}, \tilde{\alpha}_{1g}, \dots, \tilde{\alpha}_{qg})$. In addition, one may compute estimators of ARMA (p, q) parameters based on a simultaneous method-of-moments fit of the model to the second-moment matrices $\hat{V}_{(T)g}$ and $\tilde{V}_{(P)g}$ of dimensions $T \times T$ and $P \times P$, respectively. Let

$$\left(\sigma_{dg}^{*2}, \tilde{\rho}_{1g}^*, \dots, \tilde{\rho}_{pg}^*, \tilde{\alpha}_{1g}^*, \dots, \tilde{\alpha}_{qg}^* \right)$$

be the resulting vector of combined-data estimators.

5. Numerical Results

Table 1 describes the seven two-digit strata (chapters) considered in this simulation study. For a detailed description of the selected chapters, see Cho et al. (2007, Section 3.5: Selecting strata).

Table 1: Description of Selected Strata

Stratum	Stratum Description
P07	Edible vegetables, roots, and tubers
P08	Edible fruit and nuts; peel of citrus fruit or melons
P09	Coffee, tea, mate and spices
P22	Beverages, spirits, and vinegar
P61	Articles of apparel and clothing accessories
P74	Copper and articles thereof
P90	Optical, photographic, measuring and medical instruments

Figure 1 and Figure 2 show the time series plots of published STR P07 and P90 from January 1999 to December 2009, respectively. A solid red line represents the locally weighted regression smoothing predictors (loess). In locally weighted regression smoothing, the nearest neighbors of each point are used for regression and the number of neighbors is specified as a percentage of the total number of points. This

percentage is called the span and the span size used in Figure 1 and Figure 2 is 2/3. A loess predictor showed that there is not much upward or downward trend in both P07 and P90 and values are almost constant around 1.

We applied some simple time series models to the IPP short term ratio price index values using X-13 RegARIMA estimation method. RegARIMA models combine a linear regression model function with an ARIMA (autoregressive-integrated-moving average) models. Some models include a fixed or deterministic seasonal effect and we started by examining some preliminary Akaike's Information Criterion (AIC) results as in Figure 3. AIC is defined as

$$AIC = -2\hat{L} + 2 \text{ (number of model parameters)}$$

where \hat{L} is the value of the log of the likelihood at the ML estimates. Other diagnostic tools used are used: the Ljung-Box statistic, Q , which is a summary statistic reflecting general presence of autocorrelation in the residuals over lags. Q can be compared against the chi-squared distribution to check for significant residual autocorrelation; Bayesian Information Criterion (BIC); residual autocorrelation function (ACF) and partial autocorrelation function (PACF). Following are some results of log-transformed index estimates from the time span, January 1999 to December 2009.

For Chapter P07, The ARIMA (0 0 2) model seemed an adequate nonseasonal model for the P07 in terms of the standard Ljung-Box goodness-of-fit statistics. Several outliers were detected, including 4 Februarys. We observed that the ARIMA residuals contained a significant seasonal peak, an inadequacy of the model. Visually significant seasonal and trading day peaks have been found in the estimated spectrum of the RegARIMA residuals. The X-11 seasonal adjustment section indicated that there was too much variability in the series to decompose the series reliably. The ARIMA (0 0 1) with seasonal component exhibited an adequate seasonal model: a deterministic seasonal and otherwise MA(1) with a few outliers. The spectrum of the residuals from this model indicate no residual seasonality. So, this model including the seasonal component can be regarded as a reasonable model for the series P07.

Table 2.1: ARIMA(0 0 2) with no seasonal components for $\hat{\theta}_{t,g}$

parameter	estimate	se
Lag 1	0.318	0.087
Lag 2	0.006	0.087

Table 2.2: ARIMA(0 0 1) with seasonal components for $\hat{\theta}_{t,g}$

parameter	estimate	se
Lag 1	0.395	0.078

Table 2.3: Chi-Squared Tests for Groups of Regressors in ARIMA(0 0 1) with seasonal components

Regression Effect	df	Chi-square	p-value
Seasonal	11	47.09	< 0.001

For Chapter P08, at least one visually significant seasonal peak has been found in the estimated spectrum of RegARIMA residuals both in nonseasonal and seasonal models. Although the ACF of the squared residuals was improved in the seasonal model, the seasonal peak was more prominent. Further there was not much improvement in terms of AIC (-530, -542) or BIC (-509, -496) for nonseasonal and seasonal models respectively. The coefficient estimate of MA(1) was not significant in the seasonal model. Differencing once did not make much improvement in terms of Q statistics, AIC, BIC, and a seasonal peak.

Table 3.1: ARIMA(0 0 3) with no seasonal components for $\hat{\theta}_{t,g}$

parameter	estimate	se
Lag 1	0.0655	0.08445
Lag 2	0.1519	0.08384
Lag 3	0.2075	0.08458

Table 3.2: ARIMA(0 0 3) with seasonal components for $\hat{\theta}_{t,g}$

parameter	estimate	se
Lag 1	0.0683	0.08521
Lag 2	0.0573	0.08518
Lag 3	0.1050	0.08486

Table 3.3: Chi-Squared Tests for Groups of Regressors in ARIMA(0 0 3) with seasonal components

Regression Effect	df	Chi-square	p-value
Seasonal	11	41.15	< 0.001

For Chapter P90, seasonal regressors were not significant in the seasonal model. The nonseasonal model appears more adequate in terms of Q statistics, AIC, BIC, and ACF plots of residuals. MA(2) seems to fit the data better. AR models showed the similar results as MA models.

Table 4.1: ARIMA(0 0 2) with no seasonal components for $\hat{\theta}_{t,g}$

parameter	estimate	se
Lag 1	-0.0605	0.08443
Lag 2	-0.1915	0.08468

Table 4.2: ARIMA(0 0 2) with seasonal components for $\hat{\theta}_{t,g}$

parameter	estimate	se
Lag 1	-0.0841	0.08429
Lag 2	-0.1973	0.08450

Table 4.3: Chi-Squared Tests for Groups of Regressors in ARIMA(0 0 2) with seasonal components

Regression Effect	df	Chi-square	p-value
Seasonal	11	10.57	0.48

Similarly to Chapter P07, ARIMA (0 0 3) with a deterministic seasonal component exhibited an adequate seasonal model for P61. For the rest of chapters, P09, P22, and P74, a seasonal regressor was not significant. Especially for P22 and P74, a nonseasonal model seemed more adequate in terms of the standard Ljung-Box goodness-of-fit statistics as in P07. While the seasonal peak became more prominent in a seasonal model for P09, we observed an improvement in terms of the statistic of squared residuals in a seasonal model for P09. This is similar to P08.

Tables 5.1 and 5.2 respectively, report results from fitting the estimation-error covariance matrix $V_{(T)eg}$ to AR and MA models. Higher-order ARMA models were considered in preliminary work not detailed here. For the AR(1) model fits, the autoregressive coefficients were statistically significant for chapters P07 and P08, but not for the other five chapters. For the MA parameter estimation using innovation algorithm (Brockwell and Davis, 1991) showed similar results.

Table 5.1: AR(1) Parameter for e_{tg} using Yule-Walker equation

Chapter	$\hat{\phi}$	se($\hat{\phi}$)	σ^2
P07	0.080	0.032	0.003
P08	0.152	0.031	0.001
P09	-0.039	0.032	< 0.001
P22	0.004	0.032	< 0.001
P61	< 0.001	0.032	< 0.001
P74	0.019	0.032	< 0.001
P90	-0.006	0.032	< 0.001

Table 5.2: MA Parameter for e_{tg} using Innovation Algorithm

Chapter	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_3$	σ^2
P07	0.079	-0.009	-0.043	0.003
P08	0.131	0.105	0.096	< 0.001

6. Discussion

As T increases, analyses of data may encounter issues with weights that are missing or unreliable. We may consider several extensions of the paper: Although we used 12×12 covariance matrix for the analysis, in principle, we may compute an estimated covariance matrix for large values of T ; We may consider estimating time series parameters for e_{tg} based on both simulation data in $V_{(T)eg}$ and the direct bootstrap covariance estimators based on sample data; We may use estimated time series parameters for $\hat{\theta}_{tg}$ and e_{tg} to produce improved estimators of θ_{tg} per literature cited; We may apply estimated time series parameters to computation of generalized least squares estimators of the mean structure μ_{tg} e.g., deterministic month effects.

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Appendix A: Sample Design - Stratification

The IPP divided the import and export merchandise universes into two halves referred to as panels. Each panel is sampled every other year using a three stage sample design. In the IPP sample design, the first stage selects companies independently proportional to size (dollar value) within each broad product category (stratum). The second stage selects detailed product categories (classification groups) within each company using a systematic probability proportional to size (PPS) design with single random start. Each company-classification can be sampled multiple times and the number of times each company-classification is selected is then referred to as the number of quotes requested (Bobbitt et al., 2005). The raking procedure is used to adjust the second stage sample size in order to meet publication constraints across the different classification systems and to reduce respondent burden. The third stage of sampling is the selection of specific, repricable items. The field economist selects these items with the respondent's help using the disaggregation method.

Appendix B: Simulation

The IPP drew 1000 samples from an actual import panel frame, specifically from July 2002 to June 2003 import samples. This import panel included food and beverages, crude materials and related goods, vehicles and transportation equipment, and miscellaneous manufactures (Paben, 2006). The frame did not have item level information but had information on classification group and company trading dollar amounts. The IPP generated items based on the frame information of classification group and company trading dollar amounts. For company-classification groups that were selected more than once, multiple item STRs were created. To simulate price relatives for the universe, the IPP used an historical database with 13 years (from September 1993 to June 2005) worth of price relatives which were stored in combination with classification group and the month. The IPP created a universe of item STRs using the following steps: the IPP obtained the maximum number of quotes across the 1000 simulated samples for each company-classification group, and added them up for a specific classification group to get classification group quotes; then simulated price relatives from historical data using a simulation method (Chen, 2007).

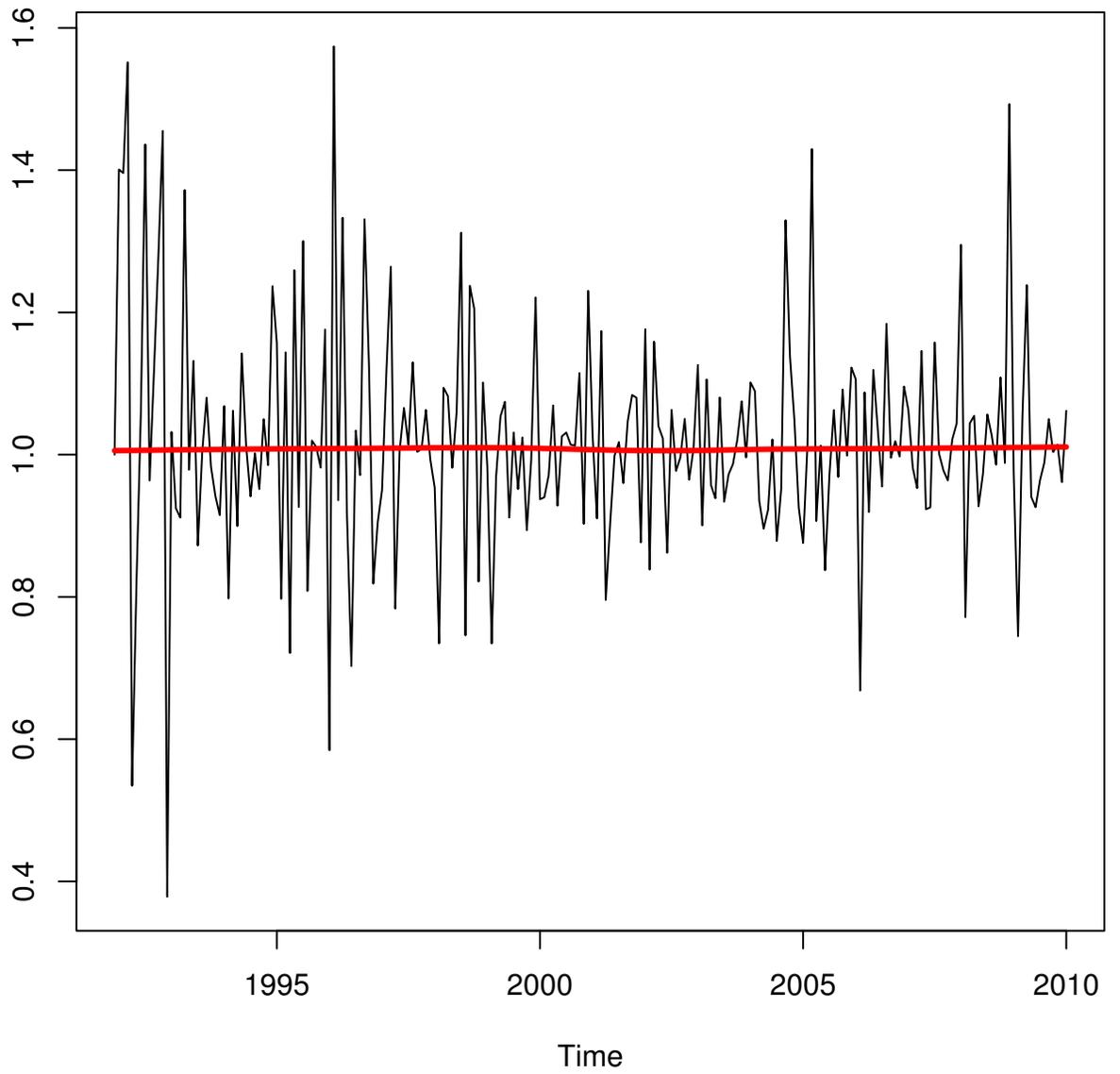


Figure 1: Published P07 STR Values from Dec 1991 to Jan 2010

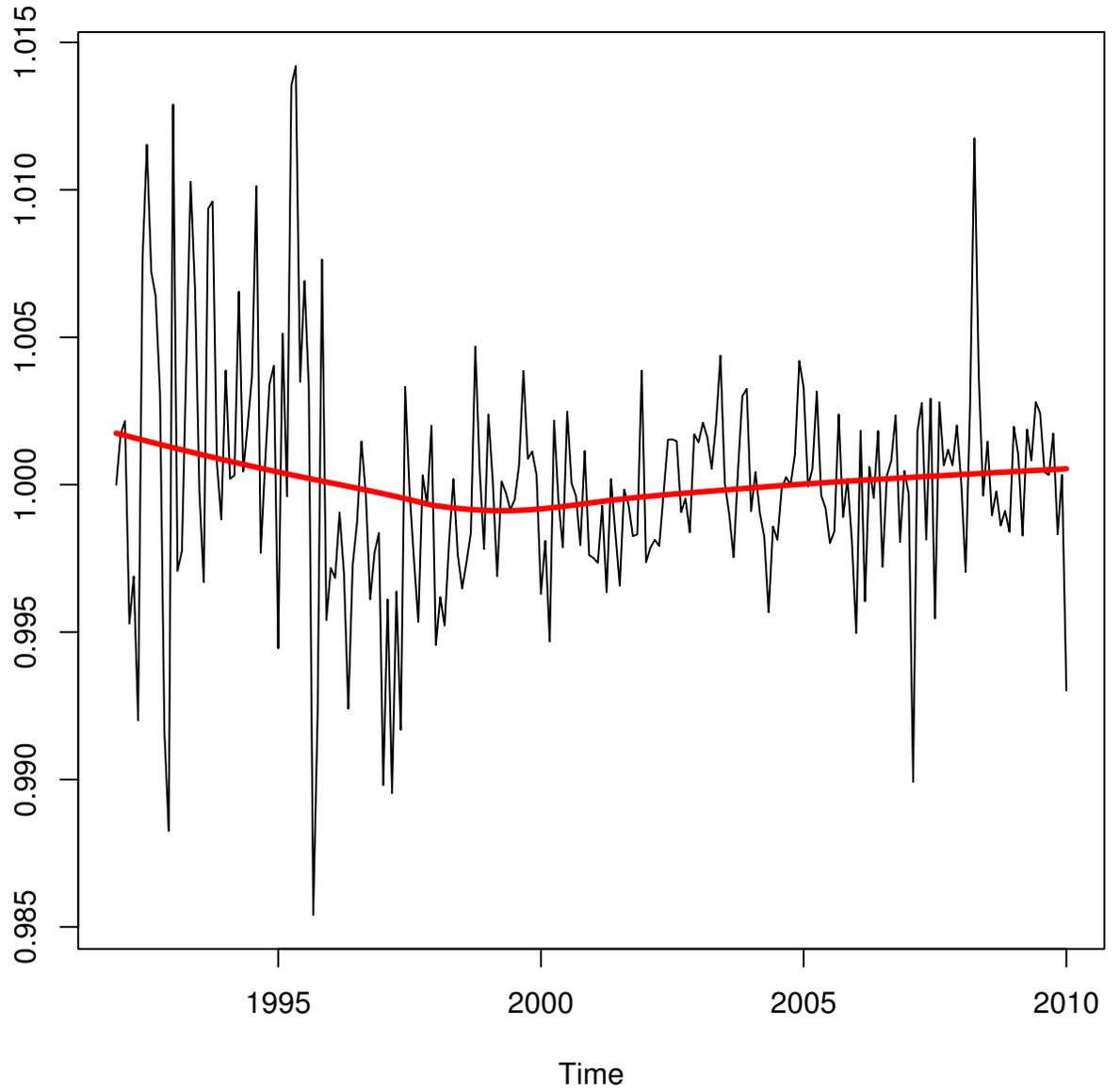


Figure 2: Published P90 STR Values from Dec 1991 to Jan 2010

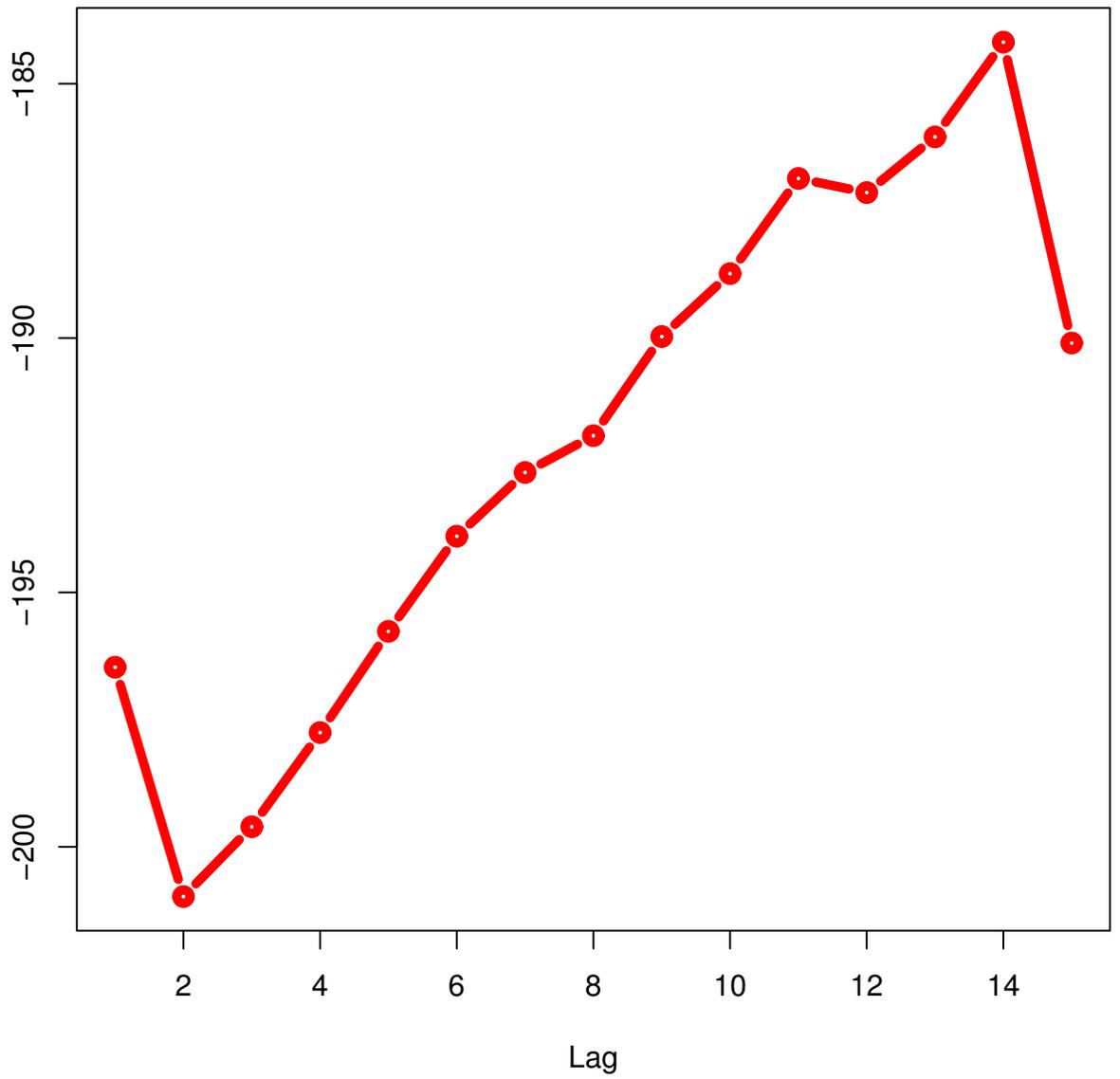


Figure 3: AIC Values for P07 STR (from Jan 1999 to Dec 2009): MA(15)