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Interarea Price Comparisons for Heterogeneous Goods and Several Levels of Commodity Aggregation

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Abstract. We derive a general form of Törnqvist multilateral (transitive) place to place index numbers and a new variant of regression methodology for imposing transitivity while minimally adjusting the initial system of bilateral index comparisons. We show that when several levels of item aggregation are to be published in a system of Törnqvist interarea parities, the adjusted, transitive Törnqvist parities at each level of aggregation preserve the aggregation rule in the unadjusted data. Finally, the method incorporates characteristics-based, hedonic quality adjustment as an integral feature. We apply the method to a subset of commodity price and expenditure data for the 44 areas of the United States covered by the Consumer Price Index. In closing, we also discuss an application of the method that makes time series and geographical comparisons consistent with one another, and note that it permits decentralization of calculation in a way that may have distinct advantages for compiling international price comparisons.

I. Introduction

Judging from periodic contacts from data users to BLS staff, place to place consumption cost comparisons for areas within the United States are very much in demand. In this paper we consider the problem of incorporating ancillary information on product characteristics to make place to place quality adjustments to interarea cost of consumption indexes. Although not specifically designed for producing interarea comparisons, the database from which the Consumer Price Index (CPI) is calculated is nevertheless a rich source of geographical price information that we exploit in this study.

We adopt a Törnqvist measurement framework, and derive a very general form for a transitive, multilateral system of parities within that framework. The Caves, Christensen, and Diewert (CCD) (1982a) implementation of the Eltetö/Köves/Szculc (EKS) methodology (described in English by Dreschler (1973)) for multilateral price measurement uses a special case of this general multilateral form, which involves the estimation of $2N - 1$ parameters when there are N items in the index aggregate. The unknown parameters represent a reference set of value shares and prices against which the shares and prices of all areas are compared. We show that, if Törnqvist aggregates are to be formed from lower level aggregates on a single, i.e. commodity, aggregation tree, the adjustment can be applied successively from the lowest to the highest levels of aggregation to produce a set of reference prices that, while fixed across areas, have components corresponding to each level of item aggregation.

We adapt earlier index number results from Zieschang (1985, 1988) and Fixler and Zieschang (1992) to incorporate information on the characteristics of the products that are systematically randomly sampled for the CPI using country-product dummy regression at "entry level item" product detail. In this index number framework, coefficients from these regressions are used in constructing quality adjustment factors for the place to place price comparisons.

We show how the parameters of the transitive Törnqvist system can be estimated with a particular regression model to impose transitivity with minimal adjustment of the data. We believe the model represents, if not a completely new approach, a substantive refinement of the regression-based multilateral adjustment that underlies the versions of EKS expounded by, for example, Cuthbert and Cuthbert (1988) and Selvanathan and Prasada Rao (1992).

Kokoski, Cardiff, and Moulton (KCM, 1994) estimate country-product dummy regressions on microdata from the U.S. Consumer Price Index for the 13 month period June 1988-July 1989. The models are fitted at the index item level to produce quality adjusted price indexes for the lowest, item level of aggregation. The regression methodology developed in this paper for imposing transitivity is demonstrated on data for a small, three-aggregation-level example problem for fruits and vegetables from an extract of 1993 data based on the KCM (1994) study, presaging the computation price of indexes for successively higher commodity aggregates for the 44 major urban centers and region-city size groups covered by the CPI.

We close by summarizing the methodological and empirical results, by describing an application of the methodology enforcing consistency between the comparisons across areas within a given time period and comparisons across time within a given area. Finally, we point out a notable advantage of this framework for compiling international parities over the narrow specification approach now used in the International Comparisons Project (ICP). Provided a standard list of item characteristics and item groups is promulgated, item strata can be broadened, increasing the likelihood of finding a useful specification from country to country. Further, this advantage is not bought at the cost of operational feasibility, since calculation can be decentralized, obviating the need for central, trans-national access to closely-guarded national micro data.

II. Economic index number concepts incorporating information on the characteristics of heterogeneous goods

Let p_i^a be the price in area *a*, of which there are *A* areas in total, of commodity *i*. Let q_i^a be the corresponding quantity purchased and let x_i^a be the vector of characteristics of the *ith* item specification transacted in area *a*. Let e_h^a represent the total expenditures of consumer unit *h* in area *a*. We will use interchangeably the terms "economic household" and "consumer unit" for the economic unit of analysis, following BLS terminology. A consumer unit is a group of individuals whose consumption decisions for significant components of expenditure are joint or shared. Let q_h^a denote the vector of goods consumed by household *h* in area *a* with vector of characteristics x_h^a and prices p_h^a.

We suppose that each consumer unit in area *a* minimizes the cost of achieving a given level of welfare at expenditure level e_h^a so that the consumer unit cost of consumption of a given quality of goods as determined by the vector x_h^a would be

$$e_h^a = E_h^a(u_h^a, x_h^a, p_h^a) = \min_{q_h^a}\left\{p_h^{a\prime} q_h^a : F_h^a\left(x_h^a, q_h^a\right) \geq u_h^a\right\}.$$

where $F_h^a\left(x_h^a, q_h^a\right)$ is the utility function of consumer unit *h* and u_h^a is the unit's welfare index.

We suppose further that the consumer unit in area *a* faces a hedonic locus of market equilibrium prices across the quality spectrum given by $p_h^a = H^a\left(x_h^a\right)$, and that the unit minimizes the cost of achieving welfare level u_h^a over the characteristics of goods, so that

$$\nabla_{x_h^a} E_h^a\left(u_h^a, x_h^a, p_h^a\right) + \nabla_{x_h^a} p_h^{a\prime} \nabla_{p_h^a} E_h^a\left(u_h^a, x_h^a, p_h^a\right) = 0 \tag{1}$$

Since $\nabla_{x_h^a} p_h^a = \nabla_{x_h^a} H^a$ and $\nabla_{p_h^a} E_h^a\left(u_h^a, x_h^a, p_h^a\right) = q_h^a$, the latter by the Shephard/Hotelling lemma, we have

$$\nabla_{x_h^a} E_h^a\left(u_h^a, x_h^a, p_h^a\right) = -\nabla_{x_h^a} H^{a\prime} q_h^a \tag{2}$$

If H^a is semilog, as generally assumed in hedonic studies, so that

$$\ln H_i^a = \alpha_i^a + \beta_i^{a\prime} x_i^a \tag{3}$$

then the characteristics gradient expression can be rewritten

$$\nabla_{x_h^a} E_h^a\left(u_h^a, x_h^a, p_h^a\right) = -\beta^{a\prime} w_h^a e_h^a \tag{4}$$

where $w_{i,h}^a = \dfrac{p_{i,h}^a q_{i,h}^a}{\sum_i p_{i,h}^a q_{i,h}^a}$

$$w_h^a = \begin{bmatrix} w_{1,h}^a \\ \vdots \\ w_{N_q,h}^a \end{bmatrix}; N_q = \text{number of commodities}$$

$$\beta^{a\prime} = \begin{bmatrix} \beta_1^{a\prime} \\ \vdots \\ \beta_{N_x}^{a\prime} \end{bmatrix}; N_x = \text{number of product characteristics}.$$

Turning now to aggregate expenditure over consumer units in an area, Diewert (1987) has considered this problem as a weighted average of individual household index numbers comparing the prices in two areas in the “democratic weighting” case. In this paper we follow his characterization of the “plutocratic” expenditure weighted case with some modifications for the heterogeneity of goods within and between areas. The area aggregate expenditure function is

$$E^a\left(\vec{u}^a, \vec{x}^a, \vec{p}^a\right) = \sum_h E_h^a\left(u_h^a, x_h^a, p_h^a\right)$$

where the $\rightarrow$ over an argument indicates the concatenation of vectors across households. We then consider the expenditure function in terms of log transformed price arguments as

$$Q^a\left(\vec{u}^a, \vec{x}^a, \ln \vec{p}^a\right) = E^a\left(\vec{u}^a, \vec{x}^a, \vec{p}^a\right) = \sum_h E_h^a\left(u_h^a, x_h^a, p_h^a\right) = \sum_h Q_h^a\left(u_h^a, x_h^a, \ln p_h^a\right).$$

We "plutocratically" aggregate across households in area *a* such that the expenditure weighted average for characteristics and log-prices represent the indicators determining area demand behavior, where area item demand is the sum of the economic household item demands for the area.[1] We do not require strong aggregation conditions, but effectively hold the distribution of product characteristics and prices fixed across economic households within area *a* as in

$$\tilde{Q}^a\left(\vec{u}^a, \bar{x}^a, \overline{\ln p}^a\right) = Q^a\left(\vec{u}^a, \iota \otimes \bar{x}^a + \nu_x^a, \iota \otimes \overline{\ln p}^a + \nu_{\ln p}^a\right) \qquad (5)$$

where

$\nu_x^a = \vec{x}^a - \iota \otimes \bar{x}^a$

$\nu_{\ln p}^a = \ln \vec{p}^a - \iota \otimes \overline{\ln p}^a$

ι = a vector of ones of dimension equal to the number of households

$\otimes$ = Kronecker product

[1] Actually, the results to follow do not depend on the particular form of area aggregation for product characteristics and prices. Although this discussion is couched in terms of an arithmetic area mean for characteristics and a geometric mean for prices, others will also work.

give the deviations of the area means from the individual household values for commodity characteristics and prices paid.

Diewert (1976) and Caves, Christensen, and Diewert (1982b) have shown, using the derivatives of the expenditure function with respect to log prices expressed in terms of observable expenditure shares, that the Törnqvist index number is exact for the Translog flexible functional form. The translog aggregator function differentially approximates any price aggregator function (i.e., cost of utility, input cost, revenue function) to the second order at a point, and it is exact for the Törnqvist index number even when some of the parameters (those on the first-order terms) of the underlying aggregator function are different in the two periods or localities compared. We take the derivative of the area expenditure function with respect to intera-area average household characteristics and price arguments to obtain

$$\frac{\partial}{\partial \bar{x}^a_{iz}} \ln \tilde{E}^a\left(\vec{u}^a, \bar{x}^a, \exp(\overline{\ln p}^a)\right) = \frac{\partial}{\partial \bar{x}^a_{iz}} \ln \tilde{Q}^a\left(\vec{u}^a, \bar{x}^a, \overline{\ln p}^a\right) = \tag{6}$$
$$= \sum_h \frac{\partial}{\partial x^a_{izh}} Q^a_h\left(u^a_h, x^a_h, \ln p^a_h\right) / \tilde{Q}^a = -\beta^a_{iz} \sum_h w^a_{ih} s^a_h = -\beta^a_{iz} \overline{w}^a_i$$

$$\frac{\partial}{\partial \overline{\ln p}^a_i} \ln \tilde{E}\left(\vec{u}^a, \bar{x}^a, \exp(\overline{\ln p}^a)\right) = \frac{\partial}{\partial \overline{\ln p}^a_i} \ln \tilde{Q}\left(\vec{u}^a, \bar{x}^a, \overline{\ln p}^a\right) \tag{7}$$
$$= \sum_h \frac{\partial}{\partial \ln p^a_{ih}} Q^a_h\left(u^a_h, x^a_h, \ln p^a_h\right) / \tilde{Q}^a = \sum_h w^a_{ih} s^q_h = \overline{w}^a_i$$

where

$$w^a_{ih} = \frac{p^a_{ih} q^a_{ih}}{\sum_i p^a_{ih} q^a_{ih}} = \frac{p^a_{ih} q^a_{ih}}{e^a_h}$$

$$s^a_h = \frac{e^a_h}{\sum_h e^a_h}$$

are, respectively, the within household expenditure shares of commodities and the between household total expenditure shares of consumer unit *h* in area *a*.

Finally, we assume that the area aggregate expenditure function $\ln \tilde{Q}^a\left(e^a, \bar{x}^a, \overline{\ln p}^a\right)$ has a quadratic, "semi-translog" functional form in its arguments with coefficients of second-order terms independent of location, but with possibly location-specific coefficients on linear terms. Following CCD (1982b), then, we can derive the following (logarithmic) index number result:

$$\ln I^{ab} =$$

$$\tfrac{1}{2}\left[\ln \tilde{Q}^a\left(\vec{u}^a, \bar{x}^b, \overline{\ln p}^b\right) - \ln \tilde{Q}^a\left(\vec{u}^a, \bar{x}^a, \overline{\ln p}^a\right) + \ln \tilde{Q}^b\left(\vec{u}^b, \bar{x}^b, \overline{\ln p}^b\right) - \ln \tilde{Q}^b\left(\vec{u}^b, \bar{x}^a, \overline{\ln p}^a\right)\right]$$

$$= \tfrac{1}{2}\left[\nabla_{\ln p} \ln \tilde{Q}^a\left(\vec{u}^a, \bar{x}^a, \overline{\ln p}^a\right) + \nabla_{\ln p} \ln \tilde{Q}^b\left(\vec{u}^b, \bar{x}^b, \overline{\ln p}^b\right)\right]\left(\overline{\ln p}^b - \overline{\ln p}^a\right) \tag{8}$$

$$+ \cdot \tfrac{1}{2}\left[\nabla_x \ln \tilde{Q}^a\left(\vec{u}^a, \bar{x}^a, \overline{\ln p}^a\right) + \nabla_x \ln \tilde{Q}^b\left(\vec{u}^b, \bar{x}^b, \overline{\ln p}^b\right)\right]\left(\bar{x}^b - \bar{x}^a\right)$$

Substituting (6) and (7) into (8), following CCD (1982b) again, and with reference to Fixler and Zieschang (1992), we have

$$\ln I^{ab} = \ln T^{ab} \equiv \tfrac{1}{2}\sum_i \left[\left(\overline{w}_i^a + \overline{w}_i^b\right)\left(\overline{\ln p}_i^b - \overline{\ln p}_i^a\right) - \sum_z \left(\beta_{iz}^a \overline{w}_i^a + \beta_{iz}^b \overline{w}_i^b\right)\left(\bar{x}_{iz}^b - \bar{x}_{iz}^a\right)\right]. \tag{9}$$

This formula for the bilateral index between areas is an extremely flexible result that permits all parameters of the semi-log "hedonic" price equations to differ by area, that fully reflects household optimization over measured product quantities and characteristics.

III. Törnqvist Multilateral (Transitive) Systems of Bilateral Index Numbers

In another paper, Caves, Christensen, and Diewert (CCD, 1982a) noted that the system of bilateral Törnqvist interarea indexes is not transitive, but developed a simply calculated multilateral variant satisfying the transitivity property. Returning to lower case notation for the index arguments for areas, we derive the following general implication of transitivity for this class of index number:

PROPOSITION 1

It is necessary and sufficient for the bilateral Törnqvist item index to be transitive, that for all *a*, *b*, there exist constant vectors w^0 and $\ln p^0$ such that

$$\sum_i w_i^a \ln p_i^b - \sum_i w^b \ln p_i^a = \sum_i w_i^0 \left(\ln p_i^b - \ln p_i^a\right) - \sum_i \ln p_i^0 (w_i^b - w_i^a) \tag{10}$$

where

w_i^0 = a reference share for index item *i* for the entire region, with $\sum_i w_i^0 = 1$.

$\ln p_i^0$ = a reference price for index item *i* across the entire region.

Furthermore, if this condition holds, the multilateral Törnqvist index has the form

$$\ln T^{ab} \equiv \sum_i \left[\tfrac{1}{2}\left(w_i^0 + w_i^b\right)\left(\ln p_i^b - \ln p_i^0\right) - \tfrac{1}{2}\left(w_i^0 + w_i^a\right)\left(\ln p_i^a - \ln p_i^0\right)\right] \quad \lozenge \tag{11}$$

The proof is given in Appendix I. CCD(1982a) showed that application of the EKS principle to a system of bilateral Törnqvist indexes yields the above formula with the reference shares and log prices set at their simple arithmetic averages across areas.

Clearly, these simple averages could also be replaced with total expenditure weighted averages. We consider still another way of estimating the reference shares and prices in the next section.

The overall system an be adjusted to be transitive in both prices and item characteristics by applying the principle underlying Proposition 1. This is stated in:

PROPOSITION 2

If the area-specific CPD coefficients are known, it is necessary and sufficient for the bilateral quality-adjusted Törnqvist item index to be transitive, that for all *a*, *b*,

$$\begin{aligned}
&\sum_n w_i^a\left(\ln p_i^b-\left(\sum_z \beta_{iz}^a x_{iz}^b\right)\right)-\left[\sum_i w_i^a\left(\ln p_i^a-\left(\sum_z \beta_{iz}^b x_{iz}^a\right)\right)\right]\\
&=\sum_i\sum_z\left[-\beta_{iz}^0 w_i^0\right]\left(x_{iz}^b-x_{iz}^a\right)+\sum_i\sum_z x_{iz}^0\left(\beta_{iz}^b w_i^b-\beta_{iz}^a w_i^a\right)\\
&+\sum_i w_i^0\left(\ln p_i^b-\ln p_i^a\right)+\sum_i\left[-\ln p_i^0\right](w_i^b-w_i^a)
\end{aligned} \tag{12}$$

where

x_{iz}^0	=	a reference characteristic *z* for index item *i* across the entire region
β_{iz}^0	=	a reference coefficient for the characteristic *z* of item *i* in a semi-log hedonic equation explaining specification price across the entire region
p_i^0	=	a reference price for item *i* across the entire region
w_i^0	=	a reference share for item *i* for the entire region.

Furthermore, if this condition holds, the bilateral Törnqvist index for item group *i* has the form

$$\ln T^{ab} = -\sum_i \sum_z \tfrac{1}{2}\left(\beta_{iz}^0 w_i^0 + \beta_{iz}^b w_i^b\right)\left(x_{iz}^b - x_{iz}^0\right) + \sum_i \tfrac{1}{2}\left(w_i^0 + w_i^b\right)\left(\ln p_i^b - \ln p_i^0\right) - \left[-\sum_i \sum_z \tfrac{1}{2}\left(\beta_{iz}^0 w_i^0 + \beta_{iz}^a w_i^a\right)\left(x_{iz}^a - x_{iz}^0\right) + \sum_i \tfrac{1}{2}\left(w_i^0 + w_i^a\right)\left(\ln p_i^a - \ln p_i^0\right)\right] \Diamond \qquad (13)$$

IV. Multilateral price measurement with subaggregates of items

Let p_{ijklmn}^a be the price in area *a*, of which there are *A* areas in total, of specification *n* in item group *m* in stratum class *l* in basic heading class *k* in group *j* in major group or division *i*. Let q_{ijklmn}^a be the corresponding quantity purchased. The bilateral Törnqvist index comparing the prices in areas *a* and *b* for item aggregate *ijklm* is

$$\ln T_{ijklm}^{ab} \equiv \sum_n \tfrac{1}{2}\left(w_{ijklmn}^a + w_{ijklmn}^b\right)\left(\ln p_{ijklmn}^b - \ln p_{ijklmn}^a\right)$$

where

$w_{ijklmn}^a =$ the value share in area *a* of specification *ijklmn* within the next-higher group *ijklm*, with

$$w_{ijklmn}^a \equiv \frac{p_{ijklmn}^a q_{ijklmn}^a}{\sum_n p_{ijklmn}^a q_{ijklmn}^a}$$

and with q_{ijklmn}^a the quantity of the specification transacted, so that

$$\sum_n w_{ijklmn}^a = 1.$$

Analysis of the contribution of subaggregates to levels of place to place indexes. In practice, index numbers are produced for hierarchical classification trees of products, industries, occupations, etc. Because Törnqvist indexes are linear in the log differences of detailed specification prices, the contribution of each subaggregate, say, women's apparel, to the all items level ratio between two areas can be readily calculated by exponentiating the appropriate weighted sums of log price differences. These sums would be calculated from the transitive expression for the index given in equation (11), where it is expressed in terms of locality weights averaged with reference weights and price differentials from reference prices. In this case, "all items" bilateral indexes are constructed as the direct aggregation of the specification prices as

$$\ln T^{ab} \equiv \sum_i \sum_j \sum_k \sum_l \sum_m \sum_n \Big[\tfrac{1}{2}\left(w^0_{ijklmn} + w^b_{ijklmn}\right)\left(\ln p^b_{ijklmn} - \ln p^0_{ijklmn}\right) - \tfrac{1}{2}\left(w^0_{ijklmn} + w^a_{ijklmn}\right)\left(\ln p^a_{ijklmn} - \ln p^0_{ijklmn}\right)\Big].$$

The contribution to the level of $\ln T^{ab}$ of major commodity group *i* would simply be the subordinate sum

$$\ln C^{ab}_i \equiv \sum_j \sum_k \sum_l \sum_m \sum_n \Big[\tfrac{1}{2}\left(w^a_{ijklmn} + w^0_{ijklmn}\right)\left(\ln p^b_{ijklmn} - \ln p^0_{ijklmn}\right) - \tfrac{1}{2}\left(w^a_{ijklmn} + w^0_{ijklmn}\right)\left(\ln p^a_{ijklmn} - \ln p^0_{ijklmn}\right)\Big].$$

The simplicity of this approach to analysis of the place to place price differentials of subaggregates, and its focus on subaggregate change within the larger "all items" context, has a great deal of appeal. The extension of this discussion to quality adjusted price indexes including characteristics is straightforward and left to the reader.

Transitivity simultaneously across several aggregation levels. Nevertheless, when place to place subaggregate indexes are to be published in addition to the "all items" index, it may not be seen as sufficient to adjust only the "all items" index to be transitive. The subaggregates would then be required not only to satisfy transitivity, but aggregate according to an index number rule to successively higher levels. This is a property distinct from consistency in aggregation, whereby an index formula for an aggregate calculated directly is the same as that calculated with the same formula successively applied to intermediate subaggregates. Rather, assuming the same index formula is repeatedly applied at each level as in the latter case, we would like all levels of aggregation to satisfy transitivity while preserving the aggregation rule, so that users might also combine low-level aggregates following the same formula and weighting and be assured of obtaining the higher level aggregates. We show that it is possible to construct such aggregation-consistent place to place indexes under a multilevel Törnqvist aggregation rule.

Having dealt with the first level of aggregation in the Section II, we now consider aggregation of the item indexes *ijklm* to the stratum level *ijkl*. We first observe from Proposition that the transitivity of the item aggregate *ijklm* permits us to identify average price levels for the aggregate for each area in the region as

$$\ln \overline{p}^{a}_{ijklm} = \sum_{n} \tfrac{1}{2}\left(w^{0}_{ijklmn} + w^{a}_{ijklmn}\right)\left(\ln p^{a}_{ijklmn} - \ln p^{0}_{ijklmn}\right)$$

allowing us to rewrite the expression for the bilateral item index as:

$$\ln T^{ab}_{ijklm} = \ln \overline{p}^{b}_{ijklm} - \ln \overline{p}^{a}_{ijklm}.$$

The bilateral index between areas *a* and *b* of the stratum aggregate *ijkl* over item groups *ijklm* is

$$\ln T^{ab}_{ijkl} = \sum_{m} \tfrac{1}{2}\left(w^{a}_{ijklm} + w^{b}_{ijklm}\right)\ln T^{ab}_{ijklm} = \sum_{m} \tfrac{1}{2}\left(w^{a}_{ijklm} + w^{b}_{ijklm}\right)\left(\ln \overline{p}^{b}_{ijklm} - \ln \overline{p}^{a}_{ijklm}\right).$$

Applying the Proposition to the stratum level, the transitive bilateral index between areas *a* and *b* of the stratum aggregate *ijkl* over item groups *ijklm* is, therefore,

$$\ln T_{ijkl}^{ab} \equiv \sum_m \Big[\tfrac{1}{2}\left(w_{ijklm}^{0} + w_{ijklm}^{b}\right)\left(\ln \overline{p}_{ijklm}^{b} - \ln p_{ijklm}^{0}\right) - \tfrac{1}{2}\left(w_{ijklm}^{0} + w_{ijklm}^{a}\right)\left(\ln \overline{p}_{ijklm}^{a} - \ln p_{ijklm}^{0}\right)\Big].$$

We note that the expression for the transitive Törnqvist *item* index above would have been obtained if the reference *specification* prices had been $\ln p_{ijklmn}^{0(m)} = \ln p_{ijklmn}^{0} + \ln p_{ijklm}^{0}$. Further, if the specification reference prices were so adjusted, the transitivity of the lower level item indexes would continue to hold. Further still, because each level's log index is the difference between weighted log price relatives, those components constant within group cancel, leaving only those elements varying with members of the group. To confirm,

$$\begin{aligned}
\ln T_{ijklm}^{ab} \equiv & \sum_n \Big[\tfrac{1}{2}\left(w_{ijklmn}^{0} + w_{ijklmn}^{b}\right)\left(\ln \overline{p}_{ijklmn}^{b} - \ln p_{ijklmn}^{0} - \ln p_{ijklm}^{0}\right) \\
& - \tfrac{1}{2}\left(w_{ijklmn}^{0} + w_{ijklmn}^{a}\right)\left(\ln \overline{p}_{ijklmn}^{a} - \ln p_{ijklmn}^{0} - \ln p_{ijklm}^{0}\right)\Big] \\
= & \sum_n \Big[\tfrac{1}{2}\left(w_{ijklmn}^{0} + w_{ijklmn}^{b}\right)\left(\ln \overline{p}_{ijklmn}^{b} - \ln p_{ijklmn}^{0}\right) \\
& - \tfrac{1}{2}\left(w_{ijklmn}^{0} + w_{ijklmn}^{a}\right)\left(\ln \overline{p}_{ijklmn}^{a} - \ln p_{ijklmn}^{0}\right)\Big] \\
& - \sum_n \Big[\tfrac{1}{2}\left(w_{ijklmn}^{0} + w_{ijklmn}^{b}\right)\ln p_{ijklm}^{0} - \tfrac{1}{2}\left(w_{ijklmn}^{0} + w_{ijklmn}^{a}\right)\ln p_{ijklm}^{0}\Big] \\
= & \sum_n \Big[\tfrac{1}{2}\left(w_{ijklmn}^{0} + w_{ijklmn}^{b}\right)\left(\ln \overline{p}_{ijklmn}^{b} - \ln p_{ijklmn}^{0}\right) \\
& - \tfrac{1}{2}\left(w_{ijklmn}^{0} + w_{ijklmn}^{a}\right)\left(\ln \overline{p}_{ijklmn}^{a} - \ln p_{ijklmn}^{0}\right)\Big].
\end{aligned}$$

In effect, then, each level of aggregation adds a component to the reference price vector, so that a system transitive at all levels of aggregation would require specification reference prices of the form

$$\ln p_{ijklmn}^{0(ijklm)} = \ln p_{ijklmn}^{0} + \ln p_{ijklm}^{0} + \ln p_{ijkl}^{0} + \ln p_{ijk}^{0} + \ln p_{ij}^{0} + \ln p_{i}^{0} .$$

Finally, only the components of the reference price vector relevant to (within) a given aggregation level enter into that level's transitivity adjustment equation. This permits a decomposition of the estimation procedure allowing the lowest aggregate reference shares and reference price components to be estimated first (using the regression equation presented in the Proposition), followed by successively higher levels in turn. Again, the extension to quality adjusted price indexes accounting for the differences in product characteristics across areas is straightforward.

More than one aggregation tree. Statistical price series are often published on more than one aggregation scheme. For example, establishment data are often published on commodities and industries (Producer Price Indexes) and occupations and industries (Employment Cost Indexes). Multiple trees can be incorporated into the structure just elucidated by merging the trees and defining cells by crossing the classification strata in the two (or more) structures. There is an new consistency issue introduced; namely, that comparable aggregates formed from differing subaggregates in the distinct classification structures should be the same. Most obviously, the "all items" price index on establishment data should be the same whether the subaggregates are industries or occupations/commodities. Similarly, the Industry Division 1 labor compensation index should be the same number, whether calculated as an aggregate of the two-digit industries or major occupational groups within Division 1. Constraints of this type bring us much closer to imposing a defacto requirement of traditional consistency in aggregation on the data, but are not equivalent to imposing the property unless each elementary price is contained in a distinct cross-cell of the two or more structures. We consider only one, commodity aggregation tree in this paper.

V. Estimation of the Reference Values for Shares, Prices, and Determinants of Quality

Adjusting for quality from place to place. Data permitting, it is standard practice in constructing place to place price indexes to adjust for known price determining specification characteristics using a regression of specification prices on measured characteristics and a set of dummy variables for locality. This country-product-dummy (CPD) approach is relatively simple to do and easily implemented. The most obvious way of controlling for quality in constructing a place to place index is to use the intercept plus coefficients on the area dummy variables as quality adjusted price levels in the bilateral price index for the item group. We show in this section that the use of the CPD model in this way is a special case of an exact Törnqvist index number that incorporates quality characteristics when there is a known hedonic function. The special case is that the hedonic function is the same from area to area, other than the intercept.

Suppose the characteristics of specification *n* in area *a* are given by the vector x^a_{ijklmn} and we define the set of dummy variables

$$L^a = \left[\text{For } b = 2,3,\cdots,A, \quad \Delta^{ab} = \begin{cases} 1 \text{ if } b = a \\ 0 \text{ otherwise} \end{cases} \right].$$

A CPD regression would be run by fitting the following model:

$$\ln p^a_{ijklmn} = \alpha^0_{ijklm} + \alpha_{ijklm}{}' L^a + \beta_{ijklm}{}' x^a_{ijklmn} + \varepsilon^a_{ijklmn}.$$

As described above, the conventional technique is to use the estimates of the area dummy parameters $\hat{\alpha}_{ijklmn}$ as the item log-prices to be used in further aggregation. Alternatively, from equation (13) the exact bilateral Törnqvist item index between areas *a* and *b* is

$$\begin{aligned}\ln T^{ab}_{ijklm} = &-\sum_n \sum_z \tfrac{1}{2}\left(\beta^{a}_{ijklmnz} w^{a}_{ijklmn} + \beta^{b}_{ijklmnz} w^{b}_{ijklmn}\right)\left(x^{b}_{ijklmnz} - x^{a}_{ijklmnz}\right) \\ &+\sum_n \tfrac{1}{2}\left(w^{a}_{ijklmn} + w^{b}_{ijklmn}\right)\left(\ln p^{b}_{ijklmn} - \ln p^{a}_{ijklmn}\right)\end{aligned}$$

where the first term is a quality adjustment and the second is the familiar price index.

If the slopes are the same across areas, as in Country-Product Dummy models, the bilateral index reduces to

$$\ln T^{ab}_{ijklm} = \sum_n \tfrac{1}{2}\left(w^{a}_{ijklmn} + w^{b}_{ijklmn}\right)\left(\ln p^{b}_{ijklmn} - \sum_z \beta_{ijklmnz} x^{b}_{ijklmnz} - \left(\ln p^{a}_{ijklmn} - \sum_z \beta_{ijklmnz} x^{a}_{ijklmnz}\right)\right)$$

which is an index number of quality-adjusted specification prices. This is equivalent to the conventional practice of using the intercept estimates for each area as the quality-corrected area price level for the specifications within the item group, since, from the CPD model, the area intercept coefficient for the item group can be expressed in terms of quality-corrected prices as

$$\alpha^{0}_{ijklm} + \alpha_{ijklm}{}' L^{a} = \ln p^{a}_{ijklmn} - \beta_{ijklm}{}' x^{a}_{ijklmn} - \varepsilon^{a}_{ijklmn}\,.$$

Although individual hedonic models by area are desirable, there may be insufficient data to obtain tight estimates of the coefficients, or to identify the coefficients at all. In the first case, noisy coefficients can be estimated more accurately by blending them with a pooled regional regression. An example of this approach is set out in Randolph and Zieschang (1987) with application to a rent model for the CPI shelter component.

The EKS/CCD Approach. CCD (1982) show that application of the unweighted Eltetö/Köves/Szulc approach to making a system of bilateral parities transitive is equivalent to choosing the reference shares and prices as

$$
\begin{aligned}
w^0_{ijklmn} &= \tfrac{1}{A}\sum_a w^a_{ijklmn} \\
\ln p^0_{ijklmn} &= \tfrac{1}{A}\sum_a \ln p^a_{ijklmn}
\end{aligned}
$$

A (preferable) weighted version would select the reference values as the weighted average across the area share in the next-higher level aggregate, as in the following for aggregation of items to strata:

$$
\begin{aligned}
w^0_{ijklmn} &= \sum_a s^a_{ijklm} w^a_{ijklmn} \\
\ln p^0_{ijklmn} &= \sum_a s^a_{ijklm} \ln p^a_{ijklmn}
\end{aligned}
$$

where

$$
s^a_{ijklm} \equiv \frac{\sum_n p^a_{ijklmn} q^a_{ijklmn}}{\sum_a \sum_n p^a_{ijklmn} q^a_{ijklmn}} .
$$

When quality adjustment information is available, the reference hedonic prices (or coefficients) and item characteristics are determined (in weighted form) by

$$
\begin{aligned}
\beta^0_{ijklmnq} w^0_{ijklmn} &= \sum_a s^a_{ijklm} \beta^a_{ijklmnq} w^a_{ijklmn} \\
x^0_{ijklmnq} &= \sum_a s^a_{ijklm} x^a_{ijklmnq} .
\end{aligned}
$$

A regression approach for minimal adjustment of the data. An alternative to (or, as noted below, a likely superclass of) the EKS/CCD approach is to apply the Proposition 1 transitivity condition directly. When this condition on the cross-weighted differences of log regional prices is not met, the data may be minimally adjusted to satisfy transitivity by fitting the following equation using least squares to obtain estimates $\left[\hat{w}^{0}_{ijklmn}, \hat{p}^{0}_{ijklmn}\right]$ for each specification *n* in item group *ijklm*:

$$w^{a}_{ijklmn} \ln p^{b}_{ijklmn} - w^{b}_{ijklmn} \ln p^{a}_{ijklmn} = w^{0}_{ijklmn}\left(\ln p^{b}_{ijklmn} - \ln p^{a}_{ijklmn}\right) - \ln p^{0}_{ijklmn}(w^{b}_{ijklmn} - w^{a}_{ijklmn}) + \varepsilon^{ab}_{ijklmn}$$

with the parameter restriction

$$\sum_{n} w^{0}_{ijklmn} = 1.$$ [2]

Recalling the *A* is the number of areas in the region, there will be at most $A(A-1)/2$ independent observations to estimate this equation for each specification *ijklmn*, and the model would be run as a stacked regression of specifications *n* within the item group *ijklm*.

In considering possible schemes for performing weighted estimation of the reference share and price paramenters, each record could be weighted by the average importance of areas *a* and *b* at the next higher level (item) aggregate; that is, by

---

[2] This restriction is not required for transitivity, but is required for aggregation consistency at the next level up, and embodies an inherent property of the solution to the variant of the transitivity functional equation leading to the reference shares and prices form for the transitive system of bilateral Törnqvist index numbers.

$$\sqrt{\tfrac{1}{2}\left(s_{ijklm}^{a}+s_{ijklm}^{b}\right)}\,.$$

In this scheme, areas with higher overall shares for the item across the region would carry more weight in determining the estimated within-item specification reference shares and prices. This is reminiscent of, if distinct from, the weighting approach suggested by Selvanathan and Prasada Rao (1992), and is more transparent as to how a weighting methodology would actually work in a system of transitive Törnqvist parities—it affects the estimates of the reference shares and prices.[3]

When quality adjustment information is available, the reference variables would be estimated in a way analogous to that for imposing transitivity in prices only as follows:

Estimate hedonic equations for each area as

$$\ln p_{ijklmn}^{a}=\alpha_{ijklm}^{a}+\beta_{ijklm}^{a\,\prime}x_{ijklmn}^{a}+\varepsilon_{ijklmn}^{a}\,.$$

Obtaining the estimates

$$\begin{bmatrix}\hat{\alpha}_{ijklm}^{a}\\ \hat{\beta}_{ijklm}^{a}\end{bmatrix}$$

for each area. Then using least squares, estimate the vector

[3] Actually, there is probably a weighting scheme for the transitivity fitting equation that generates the EKS/CCD versions of the transitive Törnqvist system of parities, but it does does not seem obvious how these weights would be determined.

$$\begin{bmatrix} x^{0}_{ijklmnq} \\ -\beta^{0}_{ijklmnq} w^{0}_{ijklmn} \\ -\ln p^{0}_{ijklmn} \\ w^{0}_{ijklmn} \end{bmatrix}$$

by fitting the equation

$$\begin{aligned} & w^{a}_{ijklm}\left(\ln p^{b}_{ijklmn} - \left(\sum_{q} \hat{\beta}^{a}_{ijklmnq} x^{b}_{ijklmn}\right)\right) - w^{a}_{ijklm}\left(\ln p^{a}_{ijklmn} - \left(\sum_{q} \hat{\beta}^{b}_{ijklmnq} x^{a}_{ijklmn}\right)\right) \\ & = \sum_{q}\left[-\beta^{0}_{ijklmnq} w^{0}_{ijklmn}\right]\left(x^{b}_{ijklmnq} - x^{a}_{ijklmnq}\right) + \sum_{q} x^{0}_{ijklmnq}\left(\hat{\beta}^{b}_{ijklmnq} w^{b}_{ijklm} - \hat{\beta}^{a}_{ijklmnq} w^{a}_{ijklm}\right) \\ & + w^{0}_{ijklmn}\left(\ln p^{b}_{ijklmn} - \ln p^{a}_{ijklmn}\right) + \left[-\ln p^{0}_{ijklmn}\right]\left(w^{b}_{ijklmn} - w^{a}_{ijklmn}\right) + \varepsilon^{ab}_{ijklmn} \end{aligned}$$

with the parameter restriction

$$\sum_{n} w^{0}_{ijklmn} = 1.$$

There will be at most $A(A-1)/2$ independent observations to estimate this equation for each specification *ijklmn*, and the model would be run as a stacked regression of specifications *n* within the item group *ijklm*. The observation weighting would follow the same scheme as in the simple case without specification characteristics and quality adjustment.

Notice that if the hedonic slope coefficients are the same across areas for each specification characteristic so that $\beta^{a}_{ijklmnq} = \beta^{b}_{ijklmnq} = \beta^{0}_{ijklmnq}$, then the estimating equation collapses to

$$w^a_{ijklm}\left(\ln p^b_{ijklmn} - \left(\sum_q \hat{\beta}^0_{ijklmnq} x^b_{ijklmn}\right)\right) - w^a_{ijklm}\left(\ln p^a_{ijklmn} - \left(\sum_q \hat{\beta}^0_{ijklmnq} x^a_{ijklmn}\right)\right)$$

$$= \left[w^0_{ijklmn}\right]\left(\ln p^b_{ijklmn} - \sum_q \hat{\beta}^0_{ijklmnq} x^b_{ijklmnq} - \left(\ln p^a_{ijklmn} - \sum_q \hat{\beta}^0_{ijklmnq} x^a_{ijklmnq}\right)\right)$$

$$+\left[-\left(\ln p^0_{ijklmn} + \sum_q \hat{\beta}^0_{ijklmnq} x^0_{ijklmnq}\right)\right](w^b_{ijklmn} - w^a_{ijklmn}) + \varepsilon^{ab}_{ijklmn}$$

In this case, the coefficient on the difference between the share vectors of the two areas is a *quality-adjusted reference price vector,* and no reference characteristics vector can be separately identified. It can, if desired, be independently determined as the EKS/CCD weighted average.

VI. An Empirical Example

An empirical example of the methodology is provided by interarea prices for U.S. urban areas derived from hedonic regressions on data from the Consumer Price Index. The CPI collects prices on a large sample of individual products, based on a probability sample of those specific products consumers are most likely to purchase in specific outlets in specific urban areas (see US DOL (1988)). This approach results in a sample which is representative of household consumption choices, but is heterogeneous in nature. For example, the category for instant coffee consists of observations on instant coffee products of different sizes, brands, caffeine content, and other characteristics. In order to compare the prices of instant coffee across cities these differences in characteristics must be explicitly accounted for, circumstances ideal for applying a hedonic regression approach. A recent major effort at BLS has produced such interarea price indices for most of the major categories of goods and services for 44 U.S. urban areas; this effort is described in detail in Kokoski, Cardiff, and Moulton (1994). A more recent application of this approach to 1993 CPI data provided the data input for this example.

The estimated hedonic regression coefficients on the areas from two expenditure classes (ECs) of food at home have been selected: EC11, fresh fruits, and EC12, fresh vegetables. These expenditure classes are represented by several item strata (IS), each of which consists of a single entry level item (ELI). The hierarchical classification scheme for this example is:

EC11: Fresh Fruits
- 1101 Apples
  - 11011 Apples
- 1102 Bananas
  - 11021 Bananas
- 1103 Oranges
  - 11031 Oranges
- 1104 Other fresh fruits
  - 11041 Other fresh fruits

EC12: Fresh Vegetables
- 1201 Potatoes
  - 12011 Potatoes
- 1202 Lettuce
  - 12021 Lettuce
- 1203 Tomatoes
  - 12031 Tomatoes
- 1204 Other fresh vegetables
  - 12041 Other fresh vegetables

In this preliminary example only the first three item strata of each EC are included and are aggregated for three areas: Philadelphia, Boston, and Pittsburgh. For purposes of exposition, we will let the lowest level of aggregation, subscripted *ijklmn*, be represented by the ELI (which, in this case, map uniquely into item strata). The next highest level, subscripted *ijklm*, is represented by the expenditure class, and these two ECs will be aggregated to a higher level ijkl which we can call “Fresh fruits and vegetables”. [This will be expanded to the full complement of item strata within these two ECs and to all 44 geographic areas in the near future.]

A log-linear hedonic regression was performed on each of these ELIs separately, as in Section VI above

$$\ln p^{a}_{ijklmn} = \alpha^{0}_{ijklm} + \alpha_{ijklm}{}' L^{a} + \beta_{ijklm}{}' x^{a}_{ijklmn} + \varepsilon^{a}_{ijklmn},$$

where $\ln p^{a}_{ijklmn}$ represents the log of the price of each item specification *n* in the *mth* ELI for the *ath* area, $x^{a}_{ijklmnz}$ are the variables defining the characteristics of each item specification, including the type of outlet where priced, and L^{a} is the dummy variable vector for area a. It has been shown (Summers [1973]) that the exponentiated coefficients α_{ijklm} are bilateral price indices for area a relative to the reference area (arbitrarily chosen as area $a = 1$). The coefficient estimates are presented in Appendix II. The expenditure shares for this simplified example of three areas and six ELIs are provided in Table 2 of Appendix III, where Wxxxxx is the share of expenditures on ELI xxxxx with respect to all three ELIs in that EC for each specific area, and S(ECx) is the share of expenditures for ECx in each area with respect to all three areas.

Employing the regression approach described in this paper for aggregation, the transitive Tornqvist indices for the simplified example are given in Table 3 of Appendix III. As would be expected, the index value for the aggregate Fresh fruits and vegetables lies between the index values for each component EC for each area. The regression coefficients obtained from the aggregation procedure are shown in Table 4, where W0(xxxxx) is the coefficient estimate w^{0}_{ijklm} for ELI xxxxx and P0(xxxxx) is the coefficient estimate p^{0}_{ijklm} for ELI xxxxx as specified in the regression equation. The regression coefficient estimates for the equation which derives the higher level of aggregation are provided in Table 5, Appendix II. Since there is only one ELI per stratum in our example, we aggregate directly to the expenditure class. Here, W0 (Ecxx) is the coefficient estimate w^{0}_{ijk} for EC xx and P0 (Ecxx) is the estimated value of p^{0}_{ijk}, as earlier specified.

For comparison, the bilateral Tornqvist indices for each EC are provided in Table 6 of Appendix III. These may be compared with the transitive parities to show, for this example, the degree of empirical adjustment required to achieve this property. Recognizing that transitivity must be achieved at the cost of characteristicity, it is useful to assess this difference (see Drechsler (1973)). In this case the magnitude of the difference between the transitive Tornqvist and its bilateral counterpart is less than 2 percent of the index value.

VII. Conclusion and an Extension

In this paper we have considered the case of a single cross section of areas within which transitive bilateral quality-adjusted price comparisons are to be made between areas. We have also considered commodity aggregation within this framework, whereby transitivity is imposed while preserving a staged Törnqvist index aggregation rule. We have applied the technique to a small subset of the commodities priced in the U.S. Consumer Price Index.

Our approach to transitivity has been a “minimum data adjustment” criterion with weighting specific to bilateral comparisons, and therefore differs from other methods of imposing transitivity in a system of bilateral place to place Törnqvist index numbers. Although our method has some appeal because we can claim to minimally purturb the data in order to impose the transitive property with weighting sensitive to specific bilateral comparisons, the area expenditure weighted sum of the log locality price levels will not necessarily be equal to zero, in contrast with the EKS/CCD approach, which satisfies this property by construction. The need for this property, as well as operational considerations such as ease of computation and calculation of measures of precision, would need to be weighed in deciding on an estimator for production of a regular statistical series of interarea price indexes. Before closing, we would like to briefly describe a promising avenue of research using this framework in a time series context.

The single chain link case. It has been a problem in the interpretation of data from the International Comparisons Project that the change in the levels of real GDP implied by the international purchasing power parities from time to time has not been the same as the growth in national GDPs measured by direct deflation using a(n implicit) time series GDP deflator. We consider here a remedy within the Törnqvist system of interarea and time series index numbers by considering a system of area indexes that are transitive both among areas within the same time period as well as between areas from differing time periods. A direct implication of this is that the index change between two periods for a given area, say *a*, can be expressed as the product of the relative level between two areas, say, *a* and *b*, in the first period, times the relative change in *b* between the two periods for any two areas *a* and *b*.

The Törnqvist item index $\ln T_{ijklm}^{ab,uv}$ between area *a* in time period *u* and area *b* in time period *v*, where $u, v \in \{t-1, t\}$, is

$$\ln T_{ijklm}^{ab,uv} = \sum_n \tfrac{1}{2}\left(w_{ijklmn}^{au} + w_{ijklmn}^{bv}\right)\left(\ln p_{ijklmn}^{bv} - \ln p_{ijklmn}^{au}\right).$$

It is straightforward to see that, for the system of between area, between period parities to be transitive, Proposition 1 applies directly in this case, with reference share and price vectors determined for the union of the two time periods and collections of areas. If quality adjustments are possible using hedonic regressions, then Proposition 2 can be applied to show the transitive form of the quality adjusted system of parities as a function of a reference share, price and hedonic coefficients vectors across areas and time periods. We note below that, under international decentralization of compilation, the country hedonic regression coefficients would generally not be the same, as in the CPD approach.

An additional comparison generally computed in this case is the change over time of the regional aggregate of areas. Examples of such indexes would be national consumer price and producer price and labor compensation indexes as composites of the subnational areas sampled to obtain the data. This index can be written as

$$\begin{aligned}\ln T^{R,uv}_{ijklm} &= \sum_a \tfrac{1}{2}\left(s^{au}_{ijklm} + s^{av}_{ijklm}\right)\ln T^{aa,uv}_{ijklm} \\ &= \sum_a \tfrac{1}{2}\left(s^{au}_{ijklm} + s^{av}_{ijklm}\right)\sum_n \tfrac{1}{2}\left(w^{au}_{ijklmn} + w^{av}_{ijklmn}\right)\left(\ln p^{av}_{ijklmn} - \ln p^{au}_{ijklmn}\right).\end{aligned}$$

By period to period and interarea transitivity

$$\ln T^{R,uv}_{ijklm} = \sum_a \tfrac{1}{2}\left(s^{au}_{ijklm} + s^{av}_{ijklm}\right)\left(\ln \overline{p}^{av}_{ijklmn} - \ln \overline{p}^{au}_{ijklmn}\right)$$

where

$$\ln \overline{p}^{au}_{ijklm} = \sum_n \tfrac{1}{2}\left(w^{00}_{ijklmn} + w^{au}_{ijklmn}\right)\left(\ln p^{au}_{ijklmn} - \ln p^{00}_{ijklmn}\right)$$

The aggregate time series index under period/area transitivity between the two periods is, therefore, a weighted average of the relative change in a set of area price levels, insuring consistency between the levels within period across area and rates of change between periods within area.

Time series/cross section transitivity over multiple periods. Clearly, the single chain link, two period case can be extended to the multiple period case by pooling the data for multiple periods. A distinct advantage of the application of this procedure is that the problem of chain drift is eliminated over the multiple period epoch being adjusted, while maintaining much of the period specificity of the weight and price components of the Törnqvist index formula. The reason is that transitivity eliminates drift, which is usually defined as the persistent deviation of a direct index between nonadjacent periods as compared with the product of adjacent period chain links covering the multiple period interval. An issue to be resolved in applying this technique is that it refers to a moving window of a fixed time duration. Data passing outside the window would not exactly satisfy the transitive property. Choosing the window as a long enough period could be expected to result in very slow change in the reference prices and shares, however, so that the effect could be minimized, at the cost of providing less of Drechsler's (1973) "characteristicity" for relatively recent time periods.

Decentralized computing of international parities while controlling for the quality of goods available in different countries. The methodology outlined here, which uses a hedonic, characteristics-based quality adjustment procedure, permits decentralized, within country estimation of the hedonic equation coefficients. This is especially attractive in view of the great, and generally justified reluctance with which most statistical offices grant access to the micro data sources of their price indexes. The prerequisite for this would be that a standard product classification would have to be adopted by all countries, and also, with each product class, a standard list of product characteristics or specification measures would have to be adopted. One such set of standards might be derived by merging of the U.S. CPI specification file, listing the characteristics measures for some 365 product categories, with a standard international commodity classification, such as the Central Product Classification or CPC of the United Nations, itself a superset of the now standard Harmonized classification for internationally traded commodities.

A compilation strategy such as this for the ICP would have a distinct advantage over the current approach of pricing a long, detailed list of narrowly specified items. The number of product strata required would be smaller, and the countries could use the estimates for their own, internal quality adjustment needs for time series and within country geographical comparisons.

VIII. Appendix I: Proofs of Propositions

Proof of Proposition 1:

The proof of this proposition follows methods used in, for example, Aczel (1966), and Eichhorn (1978). First, we establish the following solution of the *transitivity (functional) equation* for all single-valued functions *g* of two vectors of identical dimension in an argument set *D* that satisfy an identity condition $g(x,x)=0$:

$$g(x,y)+g(y,z)=g(x,z) \text{ and } g(x,x)=0,\ \forall (x,y,z)\in D$$
$$\textit{iff}$$
$$g(x,y)=h(y)-h(x).$$

Let $y=y^0$. Then for all *x* and *z* in the domain of *g*

$$g(x,y^0)+g(y^0,z)=r(x)+h(z)=g(x,z)$$

Substituting this back into the transitivity equation,

$$r(x)+h(y)+r(y)+h(z)=g(x,z).$$

By identity

$$g(y,y)=r(y)+h(y)=0$$

and hence

$$r(y) = -h(y)$$

We can now express $g(x,z)$ in terms of h as

$$g(x,z) = h(z) - h(y)$$

yielding the desired result.

From this, transitivity of the Törnqvist bilateral relative requires that, for all *a, b*

$$\ln T^{ab} = h(\vec{w}^b, \vec{p}^b) - h(\vec{w}^a, \vec{p}^a).$$

Expanding the bilateral relative expression, we have

$$\begin{aligned}\ln T^{ab} &\equiv \sum_i \tfrac{1}{2}\left(w_i^a + w_i^b\right)\left(\ln p_i^b - \ln p_i^a\right) \\ &= \tfrac{1}{2}\sum_i \left(w_i^b \ln p_i^b - w_i^b \ln p_i^b + w_i^a \ln p_i^b - w_i^b \ln p_i^a\right) \\ &= h(\vec{w}^b, \vec{p}^b) - h(\vec{w}^a, \vec{p}^a).\end{aligned}$$

We set $b = 0$ solve for $h(\vec{w}^a, \vec{p}^a)$ in terms of reference area 0 as

$$\begin{aligned}h(\vec{w}^a, \vec{p}^a) = &\left[h(\vec{w}^0, \vec{p}^0) - \tfrac{1}{2}\sum_i w_i^0 p_i^0\right] + \tfrac{1}{2}\sum_i w_i^0 \ln p_i^a - \tfrac{1}{2}\sum_i \ln p_i^0 w_i^a \\ &+ \tfrac{1}{2}\sum_i w_i^a p_i^a .\end{aligned}$$

Substituting this into the expanded equation for the transitive bilateral log parity, multiplying through by 2, and subtracting $w_i^b \ln p_i^b - w_i^a \ln p_i^a$ inside the summations from both sides, we have

$$\sum_i \left(w_i^a \ln p_i^b - w_i^b \ln p_i^a \right) = \sum_i w_i^0 \left(\ln p_i^b - \ln p_i^a \right)$$
$$- \sum_i \ln p_i^0 \left(w_i^b - w_i^a \right).$$

The expression for the transitive Törnqvist bilateral parity obtains by substituting this expression for the cross product between the area weights and prices into the expanded expression for the parity, adding and subtracting the term $\sum_i w_i^0 \ln p_i^0$, and collecting terms.

QED

Proof of Proposition 2:

The proof follows very closely that of Proposition 1. It is easy to see from this that the price level for each area now has a price and quality adjustment component.

IX. Appendix II: Hedonic Regression Results for Fruits and Vegetables

Table 1. ELI 11011: Apples

Mean of dependent variable: log price	-2.8886
Adjusted R^2	.3329

Variable	Coefficient
Area (ref=PHILADELPHIA-WILMINGTON-TRENTON, PA-DE-NJ-MD	
BOSTON-LAWRENCE-SALEM, MA-NH	-0.11302
PITTSBURGH-BEAVER VALLEY, PA	-0.07054
BUFFALO-NIAGRA FALLS, NY	-0.20282
NEW YORK CITY	0.06071
NEW YORK-CONN. SUBURBS	-0.05384
NEW JERSEY SUBURBS	-0.02993
CHICAGO-GARY-LAKE COUNTY, IL-IN-WI	0.06099
DETROIT-ANN ARBOR, MI	-0.04564
ST. LOUIS-EAST ST. LOUIS, MO-IL	-0.02697
CLEVELAND-AKRON-LORAIN, OH	-0.16546
MINNEAPOLIS-ST. PAUL, MN-WI	-0.00094
MILWAUKEE, WI	0.01006
CINCINNATI-HAMILTON, OH-KY-IN	-0.08656
KANSAS CITY, MO - KANSAS CITY, KS	-0.01494
WASHINGTON, DC-MD-VA	-0.01403
DALLAS-FORT WORTH, TX	-0.05908
BALTIMORE, MD	-0.04353
HOUSTON-GALVESTON-BRAZORIA, TX	-0.11871
ATLANTA, GA	-0.00259
MIAMI-FORT LAUDERDALE, FL	-0.03459
TAMPA-ST. PETERSBURG-CLEARWATER, FL	0.05230
NEW ORLEANS, LA	-0.02976
LOS ANGELES COUNTY, CA	-0.17093
GREATER LOS ANGELES, CA	-0.20649
SAN FRANCISCO-OAKLAND-SAN JOSE, CA	-0.19800
SEATTLE-TACOMA, WA	0.19215
SAN DIEGO, CA	-0.17869
PORTLAND-VANCOUVER, OR-WA	-0.17181
HONOLULU, HI	0.00481
ANCHORAGE, AK	-0.10670
DENVER-BOULDER, CO	-0.03535
NORTHEAST REGION, B SIZE PSUS	-0.04942
NORTHEAST REGION, C SIZE PSUS	-0.09681
NORTHEAST REGION, D SIZE PSUS	-0.08744
NORTH CENTRAL REGION, B SIZE PSUS	0.01421
NORTH CENTRAL REGION, C SIZE PSUS	-0.05190
NORTH CENTRAL REGION, D SIZE PSUS	-0.15361
SOUTH REGION, B SIZE PSUS	-0.01420
SOUTH REGION, C SIZE PSUS	-0.10217
SOUTH REGION, D SIZE PSUS	-0.04820
WEST REGION, B SIZE PSUS	-0.11932
WEST REGION, C SIZE PSUS	-0.12937

Table 1. ELI 11011: Apples

Mean of dependent variable: log price	-2.8886
Adjusted R^2	.3329

Variable	**Coefficient**
WEST REGION, D SIZE PSUS	-0.07509
Rotation group (ref=same sample as previous month)	
NEW SAMPLE	-0.03224
Month of collection (ref=JAN, 11 months of data)	
FEB	0.03022
MAR	0.04515
APR	0.05623
MAY	0.10186
JUN	0.15730
JUL	0.19332
AUG	0.20836
SEP	0.17460
OCT	0.01422
NOV	0.03296
Outlet type (ref=CHAIN GROCERY)	
INDEPENDENT GROCERY STORES	-0.07482
FULL SERVICE DEPARTMENT STORES	-0.11568
PRODUCE MARKET	-0.18148
CONVENIENCE STORES	0.05972
COMMODITY ORIENTED OUT NEC	-0.41615
OUTLET NOT ELSEWHERE CLASSIFIED	-0.41217
Package type (ref=OTHER)	
PACKAGING: LOOSE	-0.07769
PACKAGING: MULTI-PACK	-0.33425
PACKAGING: SINGLE ITEM, INDIVIDUALLY	-0.43312
Package size (ref=0-10 POUNDS)	
ABOVE 10 POUNDS	0.01042
SIZE REPRESENTS: WEIGHED ONE MULTI-PK	-0.09754
SIZE REPRESENTS: WEIGHT LABELED	-0.03956
SIZE: WEIGH 2 APPLES	-0.05256
SIZE: OTHER	OMITTED
Grade (ref=OTHER GRADE/GRADE NOT AVAILABLE)	
U.S. EXTRA FANCY	0.01337
Variety (ref=OTHER)	
DELICIOUS	0.00968
GOLDEN DELICIOUS	-0.03918
RED DELICIOUS	-0.04676
OTHER DELICIOUS	OMITTED
GRANNY SMITH	0.03385
GRAVENSTEIN	-0.17583
JONATHAN	-0.17352
MCINTOSH	-0.05280
ROME BEAUTY (RED ROME)	-0.03041
STAYMAN	-0.11321

Table 1. ELI 11011: Apples

Mean of dependent variable: log price	-2.8886
Adjusted R^2	.3329

Variable	**Coefficient**
WINESAP	0.06369
YORK (YORK IMPERIAL)	0.74756

Table 2. ELI 11021: Bananas

Mean of dependent variable: log price	-3.5118
Adjusted R^2	.3314

Variable	**Coefficient**
Area (ref=PHILADELPHIA-WILMINGTON-TRENTON, PA-DE-NJ-MD)	
BOSTON-LAWRENCE-SALEM, MA-NH	-0.01941
PITTSBURGH-BEAVER VALLEY, PA	-0.18766
BUFFALO-NIAGRA FALLS, NY	-0.01421
NEW YORK CITY	0.01522
NEW YORK-CONN. SUBURBS	-0.02748
NEW JERSEY SUBURBS	-0.00491
CHICAGO-GARY-LAKE COUNTY, IL-IN-WI	-0.02261
DETROIT-ANN ARBOR, MI	-0.22780
ST. LOUIS-EAST ST. LOUIS, MO-IL	0.04932
CLEVELAND-AKRON-LORAIN, OH	-0.14305
MINNEAPOLIS-ST. PAUL, MN-WI	-0.23664
MILWAUKEE, WI	-0.11808
CINCINNATI-HAMILTON, OH-KY-IN	-0.18154
KANSAS CITY, MO - KANSAS CITY, KS	-0.10477
WASHINGTON, DC-MD-VA	0.01819
DALLAS-FORT WORTH, TX	-0.25484
BALTIMORE, MD	-0.05988
HOUSTON-GALVESTON-BRAZORIA, TX	-0.12477
ATLANTA, GA	-0.30016
MIAMI-FORT LAUDERDALE, FL	-0.50646
TAMPA-ST. PETERSBURG-CLEARWATER, FL	-0.31348
NEW ORLEANS, LA	-0.00361
LOS ANGELES COUNTY, CA	-0.02534
GREATER LOS ANGELES, CA	-0.10411
SAN FRANCISCO-OAKLAND-SAN JOSE, CA	-0.08328
SEATTLE-TACOMA, WA	0.12307
SAN DIEGO, CA	-0.17004
PORTLAND-VANCOUVER, OR-WA	-0.12146
HONOLULU, HI	0.55898
ANCHORAGE, AK	0.43803
DENVER-BOULDER, CO	0.08929
NORTHEAST REGION, B SIZE PSUS	-0.05294
NORTHEAST REGION, C SIZE PSUS	-0.13774
NORTHEAST REGION, D SIZE PSUS	0.06198
NORTH CENTRAL REGION, B SIZE PSUS	-0.18972
NORTH CENTRAL REGION, C SIZE PSUS	-0.23027
NORTH CENTRAL REGION, D SIZE PSUS	-0.08938
SOUTH REGION, B SIZE PSUS	-0.20297
SOUTH REGION, C SIZE PSUS	-0.22290
SOUTH REGION, D SIZE PSUS	-0.04390
WEST REGION, B SIZE PSUS	-0.05745
WEST REGION, C SIZE PSUS	-0.09631
WEST REGION, D SIZE PSUS	-0.27655
Rotation group (ref=same sample as previous month)	
NEW SAMPLE	-0.01012

Table 2. ELI 11021: Bananas

Mean of dependent variable: log price	-3.5118
Adjusted R^2	.3314

Variable	**Coefficient**
Month of collection (ref=JAN, 11 months of data)	
FEB	0.09711
MAR	0.27093
APR	0.22156
MAY	0.23920
JUN	0.15363
JUL	0.12148
AUG	-0.08925
SEP	-0.04995
OCT	-0.14127
NOV	-0.05280
Outlet type (ref=CHAIN GROCERY)	
INDEPENDENT GROCERY STORES	-0.04932
FULL SERVICE DEPARTMENT STORES	-0.23667
PRODUCE MARKET	-0.13124
CONVENIENCE STORES	0.39717
COMMODITY ORIENTED OUT NEC	-0.23223
OUTLET NOT ELSEWHERE CLASSIFIED	-0.60950
Package size (ref=WEIGHED ONE BUNCH)	
SIZE REPRESENTS: WEIGHT LABELED	-0.03506
Grade (ref=OTHER GRADE/GRADE NOT AVAILABLE)	
STORE SECONDS OR OTHER THAN 1ST QUALITY	0.06409
1ST QUALITY OR CLASS	0.01581

Table 3. ELI 11031: Oranges

Mean of dependent variable: log price	-2.8848
Adjusted R^2	.3403

Variable	**Coefficient**
Area (ref=PHILADELPHIA-WILMINGTON-TRENTON, PA-DE-NJ-MD	
BOSTON-LAWRENCE-SALEM, MA-NH	0.06634
PITTSBURGH-BEAVER VALLEY, PA	0.03833
BUFFALO-NIAGRA FALLS, NY	-0.00287
NEW YORK CITY	0.13458
NEW YORK-CONN. SUBURBS	-0.02681
NEW JERSEY SUBURBS	-0.12221
CHICAGO-GARY-LAKE COUNTY, IL-IN-WI	0.23166
DETROIT-ANN ARBOR, MI	0.01161
ST. LOUIS-EAST ST. LOUIS, MO-IL	0.06972
CLEVELAND-AKRON-LORAIN, OH	0.02411
MINNEAPOLIS-ST. PAUL, MN-WI	0.07582
MILWAUKEE, WI	0.01591
CINCINNATI-HAMILTON, OH-KY-IN	0.01295
KANSAS CITY, MO - KANSAS CITY, KS	0.44553
WASHINGTON, DC-MD-VA	-0.00686
DALLAS-FORT WORTH, TX	-0.14995
BALTIMORE, MD	-0.02344
HOUSTON-GALVESTON-BRAZORIA, TX	-0.22791
ATLANTA, GA	0.11588
MIAMI-FORT LAUDERDALE, FL	-0.36500
TAMPA-ST. PETERSBURG-CLEARWATER, FL	-0.65656
NEW ORLEANS, LA	-0.02918
LOS ANGELES COUNTY, CA	-0.08845
GREATER LOS ANGELES, CA	-0.07800
SAN FRANCISCO-OAKLAND-SAN JOSE, CA	0.06305
SEATTLE-TACOMA, WA	0.05389
SAN DIEGO, CA	-0.04144
PORTLAND-VANCOUVER, OR-WA	0.10993
HONOLULU, HI	0.22252
ANCHORAGE, AK	0.35183
DENVER-BOULDER, CO	0.21348
NORTHEAST REGION, B SIZE PSUS	0.01369
NORTHEAST REGION, C SIZE PSUS	-0.03222
NORTHEAST REGION, D SIZE PSUS	0.18581
NORTH CENTRAL REGION, B SIZE PSUS	0.19250
NORTH CENTRAL REGION, C SIZE PSUS	0.09794
NORTH CENTRAL REGION, D SIZE PSUS	-0.11770
SOUTH REGION, B SIZE PSUS	0.01788
SOUTH REGION, C SIZE PSUS	-0.08665
SOUTH REGION, D SIZE PSUS	-0.06157
WEST REGION, B SIZE PSUS	0.06924
WEST REGION, C SIZE PSUS	0.04585
WEST REGION, D SIZE PSUS	-0.02118
Rotation group (ref=same sample as previous month)	
NEW SAMPLE	-0.04340

Table 3. ELI 11031: Oranges

Mean of dependent variable: log price	-2.8848
Adjusted R^2	.3403

Variable	**Coefficient**
Month of collection (ref=JAN, 11 months of data)	
FEB	0.08286
MAR	0.12605
APR	0.14488
MAY	0.14072
JUN	0.25007
JUL	0.29341
AUG	0.34215
SEP	0.39708
OCT	0.23935
NOV	-0.04562
Outlet type (ref=CHAIN GROCERY)	
INDEPENDENT GROCERY STORES	-0.10642
FULL SERVICE DEPARTMENT STORES	-0.00639
PRODUCE MARKET	-0.15877
CONVENIENCE STORES	0.01025
COMMODITY ORIENTED OUT NEC	-0.61553
OUTLET NOT ELSEWHERE CLASSIFIED	-0.95862
Package type (ref=OTHER)	
PACKAGING: MULTI-PACK	-0.19648
PACKAGING: SINGLE ITEM, INDIVIDUALLY	-0.06804
Package size	
WEIGH 2 ORANGES	-0.22778
WEIGHED 1 MULTI-PACK	-0.11644
WEIGHT LABELED	-0.10879
Grade (ref=OTHER GRADE/GRADE NOT AVAILABLE)	
U.S. FANCY	0.01961
Variety	
NAVEL	0.29912
TEMPLE	-0.11816
VALENCIA	0.14359
TANGELO	0.28032
TANGERINE	0.44601

Table 4. ELI 11041: Other fresh fruit

Mean of dependent variable: log price	-2.7285
Adjusted R^2	0.5932

Variable	**Coefficient**
Area (ref=PHILADELPHIA-WILMINGTON-TRENTON, PA-DE-NJ-MD	
BOSTON-LAWRENCE-SALEM, MA-NH	-0.03617
PITTSBURGH-BEAVER VALLEY, PA	-0.10519
BUFFALO-NIAGRA FALLS, NY	-0.20930
NEW YORK CITY	-0.01446
NEW YORK-CONN. SUBURBS	-0.02877
NEW JERSEY SUBURBS	-0.05622
CHICAGO-GARY-LAKE COUNTY, IL-IN-WI	-0.00033
DETROIT-ANN ARBOR, MI	-0.16026
ST. LOUIS-EAST ST. LOUIS, MO-IL	-0.02078
CLEVELAND-AKRON-LORAIN, OH	-0.01462
MINNEAPOLIS-ST. PAUL, MN-WI	-0.09512
MILWAUKEE, WI	-0.04899
CINCINNATI-HAMILTON, OH-KY-IN	-0.00455
KANSAS CITY, MO - KANSAS CITY, KS	-0.05681
WASHINGTON, DC-MD-VA	0.08002
DALLAS-FORT WORTH, TX	-0.03552
BALTIMORE, MD	0.03504
HOUSTON-GALVESTON-BRAZORIA, TX	-0.18318
ATLANTA, GA	0.01898
MIAMI-FORT LAUDERDALE, FL	-0.31861
TAMPA-ST. PETERSBURG-CLEARWATER, FL	-0.07724
NEW ORLEANS, LA	-0.00581
LOS ANGELES COUNTY, CA	-0.12754
GREATER LOS ANGELES, CA	-0.07642
SAN FRANCISCO-OAKLAND-SAN JOSE, CA	-0.02541
SEATTLE-TACOMA, WA	0.15074
SAN DIEGO, CA	-0.12074
PORTLAND-VANCOUVER, OR-WA	0.01383
HONOLULU, HI	0.11750
ANCHORAGE, AK	0.12539
DENVER-BOULDER, CO	0.02031
NORTHEAST REGION, B SIZE PSUS	-0.03246
NORTHEAST REGION, C SIZE PSUS	-0.13600
NORTHEAST REGION, D SIZE PSUS	0.06941
NORTH CENTRAL REGION, B SIZE PSUS	-0.03791
NORTH CENTRAL REGION, C SIZE PSUS	-0.09401
NORTH CENTRAL REGION, D SIZE PSUS	-0.20600
SOUTH REGION, B SIZE PSUS	-0.07499
SOUTH REGION, C SIZE PSUS	-0.10318
SOUTH REGION, D SIZE PSUS	-0.08453
WEST REGION, B SIZE PSUS	0.02278
WEST REGION, C SIZE PSUS	-0.05945
WEST REGION, D SIZE PSUS	-0.08199
Rotation group (ref=same sample as previous month)	
NEW SAMPLE	0.04082

Table 4. ELI 11041: Other fresh fruit

Mean of dependent variable: log price	-2.7285
Adjusted R^2	0.5932

Variable	**Coefficient**
Month of collection (ref=JAN, 11 months of data)	
FEB	-0.04646
MAR	-0.06279
APR	-0.01630
MAY	-0.01466
JUN	-0.05689
JUL	-0.14868
AUG	-0.26625
SEP	-0.23102
OCT	-0.18405
NOV	-0.13335
Outlet type (ref=CHAIN GROCERY)	
INDEPENDENT GROCERY STORES	-0.04624
FULL SERVICE DEPARTMENT STORES	0.24117
PRODUCE MARKET	-0.14419
CONVENIENCE STORES	-0.00807
COMMODITY ORIENTED OUT NEC	-0.00207
OUTLET NOT ELSEWHERE CLASSIFIED	-0.72177
Package type	
PACKAGING: MULTI-PACK	0.00157
PACKAGING: SINGLE ITEM, INDIVIDUALLY	0.04563
Package size	
WEIGH ONE MULTI-PACK	-0.39775
WEIGH 2 FRUITS	-0.31620
WEIGHT LABELED	-0.33027
Grade (ref=OTHER GRADE/GRADE NOT AVAILABLE)	
U.S. EXTRA FANCY	0.04234
Variety	
AVOCADOS	0.19300
BERRIES	0.28592
BLUEBERRIES	0.25617
CRANBERRIES	-0.02272
RASPBERRIES	0.99209
STRAWBERRIES	-0.30166
CHERRIES (SWEET/TART)	0.28835
GRAPEFRUIT	-0.90631
PINK GRAPEFRUIT	-0.03686
RED (RUBY) GRAPEFRUIT	-0.06126
WHITE (YELLOW) GRAPEFRUIT	-0.12095
GRAPES	-0.18645
RED (FLAME) SEEDLESS GRAPES	-0.00638
EMPEROR OR TOKAY GRAPES	-0.05662
REBIER GRAPES	0.04156
CONCORD GRAPES	-0.00026
THOMPSON SEEDLESS	0.05003
LEMONS	-0.26529

Table 4. ELI 11041: Other fresh fruit

Mean of dependent variable: log price	-2.7285
Adjusted R^2	0.5932

Variable	Coefficient
LIMES	-0.28323
MELONS	-0.82440
WATERMELON	-0.71551
CANTALOUPE MELONS	-0.19234
HONEYDEW MELONS	-0.01389
CASABA MELONS	-0.07582
CRENSHAW MELONS	0.29155
PERSIAN MELONS	0.26967
SANTA CLAUS MELONS	-0.08431
PEACHES	-0.44508
PEARS	-0.13922
ANJOU PEARS	-0.46646
BARTLETT PEARS	-0.45497
BOSC PEARS	-0.34758
SECKEL PEARS	0.01742
PINEAPPLES	-1.02458
PLUMS	-0.21393

Table 5. ELI 12011: Potatoes

Mean of dependent variable: log price	-3.7367
Adjusted R^2	0.6228

Variable	**Coefficient**
Area (ref=PHILADELPHIA-WILMINGTON-TRENTON, PA-DE-NJ-MD)	
BOSTON-LAWRENCE-SALEM, MA-NH	-0.06387
PITTSBURGH-BEAVER VALLEY, PA	-0.23146
BUFFALO-NIAGRA FALLS, NY	-0.07529
NEW YORK CITY	-0.11552
NEW YORK-CONN. SUBURBS	-0.08816
NEW JERSEY SUBURBS	-0.07603
CHICAGO-GARY-LAKE COUNTY, IL-IN-WI	0.15525
DETROIT-ANN ARBOR, MI	-0.20893
ST. LOUIS-EAST ST. LOUIS, MO-IL	0.17335
CLEVELAND-AKRON-LORAIN, OH	-0.15117
MINNEAPOLIS-ST. PAUL, MN-WI	-0.44411
MILWAUKEE, WI	-0.18660
CINCINNATI-HAMILTON, OH-KY-IN	-0.10455
KANSAS CITY, MO - KANSAS CITY, KS	-0.12530
WASHINGTON, DC-MD-VA	0.03654
DALLAS-FORT WORTH, TX	-0.18926
BALTIMORE, MD	-0.14338
HOUSTON-GALVESTON-BRAZORIA, TX	0.04671
ATLANTA, GA	-0.05600
MIAMI-FORT LAUDERDALE, FL	-0.15124
TAMPA-ST. PETERSBURG-CLEARWATER, FL	-0.04608
NEW ORLEANS, LA	-0.31242
LOS ANGELES COUNTY, CA	-0.06874
GREATER LOS ANGELES, CA	-0.02597
SAN FRANCISCO-OAKLAND-SAN JOSE, CA	-0.03646
SEATTLE-TACOMA, WA	-0.21794
SAN DIEGO, CA	-0.26552
PORTLAND-VANCOUVER, OR-WA	-0.19012
HONOLULU, HI	0.57928
ANCHORAGE, AK	0.31145
DENVER-BOULDER, CO	0.07387
NORTHEAST REGION, B SIZE PSUS	-0.02074
NORTHEAST REGION, C SIZE PSUS	-0.08409
NORTHEAST REGION, D SIZE PSUS	-0.00174
NORTH CENTRAL REGION, B SIZE PSUS	-0.16171
NORTH CENTRAL REGION, C SIZE PSUS	-0.18821
NORTH CENTRAL REGION, D SIZE PSUS	-0.35833
SOUTH REGION, B SIZE PSUS	-0.16423
SOUTH REGION, C SIZE PSUS	-0.11572
SOUTH REGION, D SIZE PSUS	-0.16923
WEST REGION, B SIZE PSUS	-0.27332
WEST REGION, C SIZE PSUS	-0.15643
WEST REGION, D SIZE PSUS	-0.28844
Rotation group (ref=same sample as previous month)	
NEW SAMPLE	-0.02658

Table 5. ELI 12011: Potatoes

Mean of dependent variable: log price	-3.7367
Adjusted R^2	0.6228

Variable	**Coefficient**
Month of collection (ref=JAN, 11 months of data)	
FEB	0.01009
MAR	-0.00178
APR	0.02363
MAY	0.06245
JUN	0.16776
JUL	0.17522
AUG	0.11864
SEP	0.01893
OCT	-0.07231
NOV	-0.09234
Outlet type (ref=CHAIN GROCERY)	
FULL SERVICE DEPARTMENT STORES	-0.61027
INDEPENDENT GROCERY STORES	-0.02169
PRODUCE MARKET	-0.12607
CONVENIENCE STORES	0.27450
COMMODITY ORIENTED OUT NEC	-0.39201
OUTLET NOT ELSEWHERE CLASSIFIED	-0.31633
Package type	
PACKAGING: LOOSE	0.78494
PACKAGING: MULTI-PACK, WEIGHT: GREATER T	-0.51911
PACKAGING: MULTI-PACK, WEIGHT: 0-9.999 L	0.35237
Package size	
WEIGHED 2 POTATOES	-0.32016
WEIGHT LABELED	-0.10378
Variety	
WHITE POTATO	-0.11197
ROUND OR LONG RUSSET	-0.12073
ROUND OR LONG WHITE	-0.08151
ROUND RED	0.13440
BAKING POTATO	0.07752
YAM	-0.25983
SWEET POTATO/YAM	0.28265
SWEET POTATO	-0.45422
UNABLE TO DETERMINE VARIETY	-0.40993

Table 6. ELI 12021: Lettuce

Mean of dependent variable: log price	-3.1222
Adjusted R^2	0.5385

Variable	**Coefficient**
Area (ref=PHILADELPHIA-WILMINGTON-TRENTON, PA-DE-NJ-MD	
BOSTON-LAWRENCE-SALEM, MA-NH	-0.29268
PITTSBURGH-BEAVER VALLEY, PA	-0.25850
BUFFALO-NIAGRA FALLS, NY	-0.23543
NEW YORK CITY	-0.04737
NEW YORK-CONN. SUBURBS	-0.14394
NEW JERSEY SUBURBS	-0.12854
CHICAGO-GARY-LAKE COUNTY, IL-IN-WI	-0.08079
DETROIT-ANN ARBOR, MI	-0.25233
ST. LOUIS-EAST ST. LOUIS, MO-IL	-0.10762
CLEVELAND-AKRON-LORAIN, OH	-0.36251
MINNEAPOLIS-ST. PAUL, MN-WI	-0.37324
MILWAUKEE, WI	-0.28823
CINCINNATI-HAMILTON, OH-KY-IN	0.04950
KANSAS CITY, MO - KANSAS CITY, KS	-0.19183
WASHINGTON, DC-MD-VA	-0.05585
DALLAS-FORT WORTH, TX	-0.10340
BALTIMORE, MD	-0.15619
HOUSTON-GALVESTON-BRAZORIA, TX	-0.04106
ATLANTA, GA	-0.22134
MIAMI-FORT LAUDERDALE, FL	-0.27921
TAMPA-ST. PETERSBURG-CLEARWATER, FL	-0.33078
NEW ORLEANS, LA	-0.37557
LOS ANGELES COUNTY, CA	-0.42383
GREATER LOS ANGELES, CA	-0.46923
SAN FRANCISCO-OAKLAND-SAN JOSE, CA	-0.42209
SEATTLE-TACOMA, WA	-0.31179
SAN DIEGO, CA	-0.63145
PORTLAND-VANCOUVER, OR-WA	-0.36026
HONOLULU, HI	0.11241
ANCHORAGE, AK	0.24406
DENVER-BOULDER, CO	-0.01414
NORTHEAST REGION, B SIZE PSUS	-0.15127
NORTHEAST REGION, C SIZE PSUS	-0.13739
NORTHEAST REGION, D SIZE PSUS	-0.05323
NORTH CENTRAL REGION, B SIZE PSUS	-0.25840
NORTH CENTRAL REGION, C SIZE PSUS	-0.27277
NORTH CENTRAL REGION, D SIZE PSUS	-0.31699
SOUTH REGION, B SIZE PSUS	-0.15786
SOUTH REGION, C SIZE PSUS	-0.16706
SOUTH REGION, D SIZE PSUS	-0.05474
WEST REGION, B SIZE PSUS	-0.39816
WEST REGION, C SIZE PSUS	-0.30573
WEST REGION, D SIZE PSUS	-0.43150
Rotation group (ref=same sample as previous month)	
NEW SAMPLE	-0.00552

Table 6. ELI 12021: Lettuce

Mean of dependent variable: log price	-3.1222
Adjusted R^2	0.5385

Variable	**Coefficient**
Month of collection (ref=JAN, 11 months of data)	
FEB	-0.17981
MAR	-0.27424
APR	-0.19154
MAY	-0.10586
JUN	-0.02842
JUL	-0.29127
AUG	-0.32081
SEP	-0.29942
OCT	-0.28473
NOV	0.02991
Outlet type (ref=CHAIN GROCERY)	
FULL SERVICE DEPARTMENT STORES	0.12446
INDEPENDENT GROCERY STORES	-0.02528
PRODUCE MARKET	-0.08024
CONVENIENCE STORES	0.30320
COMMODITY ORIENTED OUT NEC	-0.49838
OUTLET NOT ELSEWHERE CLASSIFIED	-0.66851
Package type	
PACKAGING: MULTIPACK	-0.12828
Package size	
SIZE REPRESENTS: WEIGHED ONE MULTI-PK	0.17900
Variety	
BIBB	0.59326
BOSTON	0.13274
BUTTERHEAD	0.91953
COS/ROMAINE	0.36190
GREEN LEAF	0.60410
RED LEAF	0.64647

Table 7. ELI 12031: Tomatoes

Mean of dependent variable: log price	-2.7011
Adjusted R^2	0.1991

Label	**Coefficient**
Area (ref=PHILADELPHIA-WILMINGTON-TRENTON, PA-DE-NJ-MD	
BOSTON-LAWRENCE-SALEM, MA-NH	0.01678
PITTSBURGH-BEAVER VALLEY, PA	-0.05929
BUFFALO-NIAGRA FALLS, NY	-0.13822
NEW YORK CITY	-0.02569
NEW YORK-CONN. SUBURBS	-0.13951
NEW JERSEY SUBURBS	-0.00121
CHICAGO-GARY-LAKE COUNTY, IL-IN-WI	-0.05506
DETROIT-ANN ARBOR, MI	-0.12991
ST. LOUIS-EAST ST. LOUIS, MO-IL	-0.24150
CLEVELAND-AKRON-LORAIN, OH	-0.33654
MINNEAPOLIS-ST. PAUL, MN-WI	-0.19602
MILWAUKEE, WI	-0.30839
CINCINNATI-HAMILTON, OH-KY-IN	-0.04369
KANSAS CITY, MO - KANSAS CITY, KS	-0.15001
WASHINGTON, DC-MD-VA	0.00536
DALLAS-FORT WORTH, TX	-0.27754
BALTIMORE, MD	-0.09699
HOUSTON-GALVESTON-BRAZORIA, TX	-0.10590
ATLANTA, GA	-0.05248
MIAMI-FORT LAUDERDALE, FL	-0.50285
TAMPA-ST. PETERSBURG-CLEARWATER, FL	-0.16919
NEW ORLEANS, LA	-0.14505
LOS ANGELES COUNTY, CA	-0.32307
GREATER LOS ANGELES, CA	-0.39316
SAN FRANCISCO-OAKLAND-SAN JOSE, CA	-0.25084
SEATTLE-TACOMA, WA	-0.29454
SAN DIEGO, CA	-0.52545
PORTLAND-VANCOUVER, OR-WA	-0.17875
HONOLULU, HI	0.04541
ANCHORAGE, AK	0.13065
DENVER-BOULDER, CO	-0.11228
NORTHEAST REGION, B SIZE PSUS	-0.06827
NORTHEAST REGION, C SIZE PSUS	-0.13375
NORTHEAST REGION, D SIZE PSUS	0.19425
NORTH CENTRAL REGION, B SIZE PSUS	-0.12224
NORTH CENTRAL REGION, C SIZE PSUS	-0.22069
NORTH CENTRAL REGION, D SIZE PSUS	-0.24259
SOUTH REGION, B SIZE PSUS	-0.20441
SOUTH REGION, C SIZE PSUS	-0.28840
SOUTH REGION, D SIZE PSUS	-0.21121
WEST REGION, B SIZE PSUS	-0.31270
WEST REGION, C SIZE PSUS	-0.28317
WEST REGION, D SIZE PSUS	-0.46059
Rotation group (ref=same sample as previous month)	
NEW SAMPLE	-0.02653

Table 7. ELI 12031: Tomatoes

Mean of dependent variable: log price	-2.7011
Adjusted R^2	0.1991

Label	**Coefficient**
Month of collection (ref=JAN, 11 months of data)	
FEB	-0.06562
MAR	0.03005
APR	0.24313
MAY	0.38750
JUN	0.54470
JUL	0.20950
AUG	-0.19194
SEP	-0.18685
OCT	-0.25130
NOV	-0.12365
Outlet type (ref=CHAIN GROCERY)	
FULL SERVICE DEPARTMENT STORES	-0.19366
INDEPENDENT GROCERY STORES	-0.05455
PRODUCE MARKET	-0.25519
CONVENIENCE STORES	0.57844
COMMODITY ORIENTED OUT NEC	-0.73586
OUTLET NOT ELSEWHERE CLASSIFIED	-0.69563
Package type	
PACKAGING: MULTI-PACK	0.16951
PACKAGING: SINGLE ITEM, INDIVIDUALLY	0.48149
Package size	
SIZE REPRESENTS: WEIGHED 2 TOMATOES	-0.14385
Variety	
UNSPECIFIED VARIETY	-0.05647
FIELD GROWN/VINE-RIPENED	-0.30496
HOT HOUSE OR GREENHOUSE	-0.26962
UNABLE TO DETERMINE TYPE	-0.26013

Table 8. ELI 12041: Other vegetables

Mean of dependent variable: log price	-3.0502
Adjusted R^2	0.4872

Variable	**Coefficient**
Area (ref=PHILADELPHIA-WILMINGTON-TRENTON, PA-DE-NJ-MD	
BOSTON-LAWRENCE-SALEM, MA-NH	-0.18035
PITTSBURGH-BEAVER VALLEY, PA	-0.19013
BUFFALO-NIAGRA FALLS, NY	0.16605
NEW YORK CITY	0.08782
NEW YORK-CONN. SUBURBS	-0.11219
NEW JERSEY SUBURBS	-0.07710
CHICAGO-GARY-LAKE COUNTY, IL-IN-WI	0.07671
DETROIT-ANN ARBOR, MI	-0.23765
ST. LOUIS-EAST ST. LOUIS, MO-IL	0.04041
CLEVELAND-AKRON-LORAIN, OH	-0.25811
MINNEAPOLIS-ST. PAUL, MN-WI	-0.06638
MILWAUKEE, WI	-0.10271
CINCINNATI-HAMILTON, OH-KY-IN	-0.04813
KANSAS CITY, MO - KANSAS CITY, KS	-0.01079
WASHINGTON, DC-MD-VA	0.05914
DALLAS-FORT WORTH, TX	-0.03080
BALTIMORE, MD	-0.13070
HOUSTON-GALVESTON-BRAZORIA, TX	-0.21465
ATLANTA, GA	-0.16305
MIAMI-FORT LAUDERDALE, FL	-0.15624
TAMPA-ST. PETERSBURG-CLEARWATER, FL	-0.16947
NEW ORLEANS, LA	-0.12714
LOS ANGELES COUNTY, CA	-0.30328
GREATER LOS ANGELES, CA	-0.30388
SAN FRANCISCO-OAKLAND-SAN JOSE, CA	-0.25710
SEATTLE-TACOMA, WA	0.28713
SAN DIEGO, CA	-0.41637
PORTLAND-VANCOUVER, OR-WA	-0.41273
HONOLULU, HI	0.52045
ANCHORAGE, AK	0.34271
DENVER-BOULDER, CO	0.04067
NORTHEAST REGION, B SIZE PSUS	-0.09467
NORTHEAST REGION, C SIZE PSUS	-0.23278
NORTHEAST REGION, D SIZE PSUS	-0.12643
NORTH CENTRAL REGION, B SIZE PSUS	-0.18928
NORTH CENTRAL REGION, C SIZE PSUS	-0.10230
NORTH CENTRAL REGION, D SIZE PSUS	-0.33089
SOUTH REGION, B SIZE PSUS	-0.11132
SOUTH REGION, C SIZE PSUS	-0.15629
SOUTH REGION, D SIZE PSUS	-0.06746
WEST REGION, B SIZE PSUS	-0.38406
WEST REGION, C SIZE PSUS	-0.13431
WEST REGION, D SIZE PSUS	-0.21889
Rotation group	
NEW SAMPLE	-0.07898

Table 8. ELI 12041: Other vegetables

Mean of dependent variable: log price	-3.0502
Adjusted R^2	0.4872

Variable	Coefficient
Month of collection (ref=JAN, 11 months of data)	
FEB	-0.04138
MAR	-0.06361
APR	0.04834
MAY	-0.03054
JUN	0.01922
JUL	-0.07667
AUG	-0.14672
SEP	-0.17824
OCT	-0.18042
NOV	-0.10289
Outlet type (ref=CHAIN GROCERY)	
FULL SERVICE DEPARTMENT STORES	0.02692
INDEPENDENT GROCERY STORES	-0.03887
PRODUCE MARKET	-0.12645
CONVENIENCE STORES	0.21838
COMMODITY ORIENTED OUT NEC	-0.35484
OUTLET NOT ELSEWHERE CLASSIFIED	-0.55401
Package type	
PACKAGING: MULTI-PACK	0.36462
TRIMMED	-0.02206
Package size	
WEIGHED ONE MULTI-PACK	-0.44977
Variety	
RADISHES WITH TOPS	-0.01645
RADISHES WITHOUT TOPS	-0.76580
YELLOW CORN	-0.50855
WHITE CORN	-0.13763
ARTICHOKES	0.56677
ASPARAGUS	0.58202
BEAN SPROUTS	0.04937
MINIATURE CARROTS	0.11278
GREEN SNAP BEANS	-0.01606
POLE BEANS	-0.05602
YELLOW WAX BEANS	0.42235
LIMA BEANS	0.60288
DOMESTIC (GREEN) CABBAGE	-1.01275
SAVOY (CRINKLED LEAF) CABBAGE	-0.68162
CHINESE (CELERY) CABBAGE	-0.12622
HEARTS OF CELERY	-0.02623
YELLOW ONIONS	-0.95681
WHITE ONIONS	-0.38899
PICKLING CUCUMBERS	-0.04579
SPAGHETTI SQUASH	-0.49575
YELLOW STRAIGHTNECK SQUASH	-0.08479
YELLOW CROOKNECK SQUASH	0.06090
BUTTERNUT SQUASH	-0.31871

Table 8. ELI 12041: Other vegetables

Mean of dependent variable: log price	-3.0502
Adjusted R^2	0.4872

Variable	**Coefficient**
ACORN SQUASH	-0.29270
ZUCCHINI (ITALIAN) SQUASH	-0.11483
GREEN PEPPERS	0.20108
REGULAR MUSHROOMS	-0.27425
SPANISH ONION	-0.52453
RED ONION	-0.11692

X. Appendix III: Sample Index Calculation for Fruits and Vegetables

Table 1: CPD results for Bilateral relatives for Fruits and vegetables Entry Level Items

ELI	Description	AREA1 (PHILA)	AREA2 (BOSTON)	AREA3 (PITTSBG)
11011	Apples	1.0000	0.89618	0.96884
11021	Bananas	1.0000	0.98656	0.83609
11031	Oranges	1.0000	1.04715	1.05667
12011	Potatoes	1.0000	0.95105	0.81712
12021	Lettuce	1.0000	0.78312	0.75877
12031	Tomatoes	1.0000	1.03136	0.95217

Table 2: Expenditure shares within and across areas

Share Type	AREA1(PHILA)	AREA2 (BOSTON)	AREA3 (PITTSBG)
		ELI shares by area	
W11011	0.39638	0.40456	0.52877
W11021	0.33412	0.27270	0.31936
W11031	0.26950	0.32273	0.15187
W12011	0.41063	0.36953	0.35684
W12021	0.29234	0.34244	0.37069
W12031	0.29704	0.28803	0.27247
		Expenditure class shares by area	
S(EC11)	0.46262	0.33328	0.20410
S(EC12)	0.42243	0.38425	0.19331

Table 3: Multilateral Törnqvist indexes

Item	TORN12	TORN13	TORN21	TORN23	TORN31	TORN32
EC11	0.96610	0.93711	1.03509	0.97000	1.06711	1.03093
EC12	0.90929	0.82722	1.09976	0.90974	1.20877	1.09921
EC11+EC12	0.93193	0.88096	1.07305	0.94531	1.13513	1.05786

Table 4: Estimated reference shares and prices at the ELI level

EC	ELI	Variable:	Coefficient Estimate:
Fruits	Apples	W0 (11011)	0.4391
	Bananas	W0 (11021)	0.3094
	Oranges	W0 (11031)	0.2515
	Apples	P0 (11011)	0.0458
	Bananas	P0 (11021)	0.0585
	Oranges	P0 (11031)	-0.0318
		Adjusted R^2 = 0.9708	
Vegetables	Potatoes	W0 (12011)	0.3803
	Lettuce	W0 (12021)	0.3331
	Tomatoes	W0 (12031)	0.2866
	Potatoes	P0 (12011)	0.0780
	Lettuce	P0 (12021)	0.1671
	Tomatoes	P0 (12031)	0.0041
		Adjusted R^2 = 0.9980	

Table 5: Estimated reference expenditure shares and prices at the Expenditure class level

EC	Variable:	Coefficient Estimate:
Fruits	W0 (EC11)	0.5041
	P0 (EC11)	0.0609
Vegetables	W0 (EC12)	0.4959
	P0 (EC12)	0.1735
	Adjusted R^2 = 0.9947	

Table 6: Unadjusted bilateral Törnqvist indexes

EC	TORN12	TORN13	TORN21	TORN23	TORN31	TORN32
EC 11	0.96622	0.94033	1.03496	0.98959	1.06346	1.01052
EC 12	0.91564	0.83279	1.09213	0.91505	1.20078	1.09284

XI. Appendix IV. Fruits and Vegetables Calculation for 44 CPI Areas

Table 1: Regression Coefficients for EC11:

Variable	Coefficient Estimate
W0_11011	0.1759
P0_11011	0.0530
W0_11021	0.1697
P0-11021	0.0871
W0-11031	0.0881
P0_11031	0.0014
W0_11041	0.5663
P0_11041	0.0449
Restriction	-0.000095

Adjusted R-square= 0.9832

Table 2: Regression Coefficients for EC12

Variable	Coefficient Estimate
W0_12011	0.1470
P0_12011	0.0951
W0_12021	0.1268
P0_12021	0.2001
W0_12031	0.1316
P0_12031	0.1704
W0_12041	0.5946
P0_12041	0.1219
Restriction	0.000287

Adjusted R-squared= 0.9918

Table 3: Regression Coefficients for Fresh Fruits and Vegetables

Variable	Coefficient Estimate
W0_11	0.5209
P0_11	0.0939
W0_12	0.4791
P0_12	0.2637
Restriction	-3.3956 E-15

Adjusted R-squared= 0.9942

Table 3: Fruits and Vegetables Törnqvist Indexes for 44 Areas

Area	Area Name	EC11	EC12	Both
1	Philadelphia	1.000	1.000	1.000
2	St. Louis	0.999	1.006	1.005
3	Cleveland	0.975	0.797	0.870
4	Minn-St Paul	0.922	0.828	0.890
5	Milwaukee	0.980	0.856	0.921
6	Cincinnati	0.962	0.957	0.960
7	Kansas City	1.011	0.912	0.966
8	Washington	1.050	1.031	1.043
9	Dallas-Ft Wort	0.912	0.884	0.893
10	Baltimore	0.989	0.863	0.927
11	Houston	0.831	0.856	0.834
12	Boston	0.979	0.839	0.898
13	Atlanta	0.967	0.853	0.898
14	Miami	0.743	0.795	0.764
15	Tampa	0.857	0.857	0.852
16	New Orleans	0.986	0.822	0.895
17	LA County	0.909	0.753	0.832
18	Greater LA	0.898	0.736	0.817
19	San Francisco	0.936	0.775	0.852
20	Seattle	1.165	1.062	1.109
21	San Diego	0.863	0.641	0.745
22	Portland OR	0.962	0.689	0.817
23	Pittsburgh	0.921	0.824	0.870
24	Honolulu	1.196	1.391	1.279
25	Anchorage	1.175	1.319	1.237
26	Denver	1.048	1.012	1.030
27	NE B-size	0.979	0.923	0.942
28	NE C-size	0.903	0.820	0.869
29	NE D-size	1.055	0.966	1.004
30	NC B-size	0.975	0.840	0.905
31	NC C-size	0.928	0.860	0.896
32	NC D-size	0.856	0.751	0.799
33	South B-size	0.930	0.873	0.903
34	Buffalo	0.875	1.033	0.945
35	South C-size	0.881	0.845	0.853
36	South D-size	0.925	0.909	0.902
37	West B-size	0.969	0.711	0.836
38	West C-size	0.947	0.822	0.881
39	West D-size	0.889	0.729	0.805
40	New York City	1.021	1.017	1.018
41	New York- CT	0.965	0.885	0.918
42	New York-NJ	0.947	0.910	0.924
43	Chicago	1.041	1.056	1.046
44	Detroit	0.887	0.806	0.849

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